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DEDUCTIVE LOGIC

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THE ELEMENTS

OF

DEDUCTIVE LOGIC

DESIGNED MAINLY
FOR THE USE OF JUNIOR STUDENTS
IN THE UNIVERSITIES

BY

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PREFACE.

THE precise object of the following pages is (without pre-supposing any technical acquaintance with logical terminology) to enable a student of average intelligence to acquire for himself an elementary knowledge of the main problems, principles, and rules of Deductive Logic. They are not designed to save him the trouble of afterwards consulting more advanced text-books, either in his own or other languages. The English student who wishes to gain an exact and detailed knowledge of the relations of Deduction to Induction, and consequently of the true place and value of the former process in any special science, must still have recourse to the works of Mr. Mill; or, if he wish to trace the history of logical terms and doctrines (one of the most important chapters in the history of both ancient and modern literature), he must still consult Sir W. Hamilton's *Lectures*, and the Appendices and Notes of Dr. Mansel to Aldrich's *Logic*.

To these works, as well as to Archbishop Whately's luminous *Chapter on Fallacies*, and to the original and suggestive work of Mr. James Mill on the *Analysis of the Phenomena of the Human Mind*, the Author must, once for all, express his obligations. He has, however, endeavoured, on all disputed points, to reason out his

place of the more ordinary forms Definition, Division, Classification, &c. The object of this change is to suggest to the student the importance of distinguishing the *results* from the *processes* by which they are gained. Many words employed in Logic and Psychology admit of both these meanings, and it is only by prefixing the indefinite article or using the plural number, as when we speak of 'a definition' or 'definitions,' that we can make it plain that we mean the result and not the process. It would be very difficult in all cases to mark the distinction, but I have endeavoured to do so wherever it seemed to be of any importance[1].

[1] A similar confusion in many of the terms employed in Physics has been noticed by Mr. [now Sir W. R.] Grove (*Correlation of Forces*, fifth ed., p. 251). 'Another confusion of terms has arisen, and has, indeed, much embarrassed me in enunciating the propositions put forth in these pages on account of the imperfection of scientific language; an imperfection in great measure unavoidable, it is true, but not the less embarrassing. Thus, the words light, heat, electricity, and magnetism, are constantly used in two senses—viz. that of the force producing, or the subjective idea of force or power, and of the effect produced, or the objective phenomenon. The word motion, indeed, is only applied to the effect, and not to the force, and the term chemical affinity is generally applied to the force, and not to the effect; but the other four terms are, for want of a distinct terminology, applied indiscriminately to both.'

PREFACE TO SUBSEQUENT EDITIONS.

IN subsequent Editions several minor alterations and additions have been introduced into the Text of some of the Chapters. A few Notes, as well as an Index and a Collection of Examples, have also been added. In making these alterations and additions, the Author has gladly availed himself of suggestions kindly offered, from time to time, by various correspondents and friends, amongst whom he ought specially to mention Professor Park, of Belfast. He has, however, endeavoured to prevent the work from materially exceeding the limits of its original size.

Several friends have suggested to the Author the introduction of two new chapters, one on the Categories, the other on the Formation of Terms. Influenced partly by a desire not to increase the bulk of the volume, and partly by still more important reasons, he has, after some hesitation, decided against the introduction of these chapters. The doctrine of Categories is important in the history of Logic[1], but it is not properly a branch of Logic, as that word is now generally understood and as it has been understood throughout this work. The Formation of Terms is a subject which properly belongs to Psychology and not to Logic, and moreover could not be adequately treated in a small compass. It is true that some questions properly belonging to Psychology or the History of

[1] On the categories of Aristotle the reader is referred to Hamilton's *Lectures on Logic*, Lect. xi., and the Appendix to Mansel's *Aldrich*, Note B. Both Sir W. Hamilton and Dr. Mansel, especially the former, insist on their 'wholly extra-logical' character.

Logic have been noticed in various parts of the book, but they have only been casually alluded to, not treated in distinct chapters. Logic has always been over-weighted with extraneous matter, and, wherever it is possible, it is desirable to relieve it of its superfluities, though much discretion may be needed in the process, and though the requirements of examinations have a constant tendency to lead writers on Logic to consider not how little, but how much they can introduce into their works.

In the Fifth and subsequent Editions, the Author has added an Appendix on the Heads of Predicables. There is no branch of elementary Logic so difficult to state in a form at once satisfactory to the teacher and intelligible to the learner. The attempt at a scientific treatment which he has made in Pt. II. ch. v. is, he believes, usually found too complicated for a beginner. As, however, he cannot state his own view of the doctrine in a simpler form, he has thought it the best course to add an Appendix, which, without any attempt at an exhaustive treatment, simply offers an explanation of the five words, Genus, Species, Differentia, Property, Accident. This Appendix, on the first reading of the book, may be substituted by the student for Pt. II. ch. v., but should be combined with the formal definitions given on p. 45, of which, in fact, it furnishes an explanation.

The principal changes in the eighth Edition are the addition of notes to Pt. III. chs. i. and vii., and some alterations in the text of the latter part of ch. vii.; and, in the present Edition, certain additions to the latter part of ch. vii. and to the foot-note on p. 73.

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ELEMENTS

OF

DEDUCTIVE LOGIC

INTRODUCTION.

CHAPTER I.

On the Relation of Logic to Psychology.

PSYCHOLOGY, or Mental Philosophy, may be defined as the science which classifies and analyses the phenomena of the human mind. It notes mental phenomena, considers them in their mutual relations, and investigates the mode of their generation. Of these mental phenomena some are called emotional, others intellectual. The intellectual phenomena may be regarded as the result, partly of Perception, partly of Imagination, partly of Comparison, Reflexion, or Thought. Perception is the act or operation of apprehending some present phenomenon. Imagination is the act or operation of representing to the mind some absent phenomenon. Comparison, Reflexion, or Thought, is the act or operation of comparing phenomena, whether present or absent, as well as of comparing, either with these or with one another, the results themselves which we derive from such comparisons. Thus I perceive this particular rose before me, its colour, its smell, and the pleasure I derive

from seeing and smelling it. All these perceptions I can recall to-morrow, even though the rose be absent, by the act or operation of imagination. Lastly, I may compare this particular rose with another which lies on the table, or with one which I saw yesterday, and I may express their similarity by calling them both moss-roses, or their difference by calling one a moss-rose and the other a Tudor rose. Again, moss-rose and Tudor rose, which names are both results of the act or operation of comparison, may themselves be compared, and their points of similarity expressed by the word 'rose.' So rose and dahlia may be compared, and their points of similarity expressed by the word 'flower.' Or I may compare the feeling with which I contemplate the rose with similar feelings which I have previously experienced, and call it 'pleasure'; or I may compare it with the feeling which I experience when I prick my finger with a thorn, and call one 'pleasure,' the other 'pain'; or I may compare pleasure and pain themselves, and call them both 'feelings.' The act of making comparisons, and of apprehending similarities and differences, is usually called Thought or Thinking, and the results at which it arrives Thought or Thoughts. The act or operation itself, as distinguished from other mental acts or operations, and the results which ensue from its correct or incorrect exercise, are alike legitimate subjects of investigation for the psychologist or mental philosopher. But the more detailed consideration of the latter, i.e. Thoughts or the results of Thinking, becomes the subject

of a science with a distinct name, Logic, which is thus a subordinate branch of the wider science, Psychology.

Note 1.—The term Perception is here used in its ordinary sense. The distinctions between External and Internal Perception, Perception Proper and Sensation Proper, are foreign to the present subject. They are discussed at great length in the works of Sir William Hamilton and Dr. Mansel, as well as in those of the Scottish school of metaphysicians generally, but, as involving some of the most abstruse and disputed questions in Psychology, it is not necessary or desirable that the student should acquaint himself with them till he commences the special study of that science.

Note 2.—Imagination, as here defined, is what may be called Simple, or Reproductive, as distinguished from what may be called Complex, or Productive Imagination. The former simply represents to the mind absent objects of perception as they have already been perceived, the latter *combines* phenomena or portions of phenomena, whether absent or present, into a new whole. Thus the notions of a particular man or a particular horse, if the man and horse be absent, are products of simple or reproductive imagination; the notion of a centaur would be a product of complex or productive imagination. For further and more precise information on this distinction, which is here necessarily stated somewhat roughly,

see Sir W. Hamilton's *Lectures on Metaphysics*, Lect. xxxiii. The creations of poetry and art are results of complex imagination, or, in other words, of repeated processes of simple imagination and comparison. Inasmuch as complex imagination may be analysed into simple imagination and comparison, and is thus not one of the ultimate acts or operations to which our mental phenomena are traceable, it would have been beside my purpose to have noticed it in the text.

Note 3.—I have employed the expression 'act or operation,' avoiding the expression 'power or faculty,' as the latter implies a theory of mental phenomena which would be rejected by many psychologists. It has been necessary to speak of 'act *or* operation,' as the word 'act,' like many other terms of logic or psychology, may mean either the operation or the result. This is an instance at the very outset of the ambiguity noticed in the paragraph at the end of the preface.

CHAPTER II.

Definition of Logic.

IT is the province of Logic to distinguish correct from incorrect thoughts, i. e. to analyse those thoughts which are accepted by mankind as indubitably correct, and to point out wherein they differ from those which are regarded as doubtful or incorrect; and, as a consequence of this function, it is also its province to lay down rules for the attainment of correct thoughts and for the avoidance of incorrect thoughts. Thus Logic is both a Science and an Art. It is a Science, inasmuch as it furnishes us with *a knowledge of what is*, inasmuch as it is an analysis, and determines the conditions on which valid thoughts depend. It is an Art, inasmuch as it lays down *rules for practice*, and thus enables us to detect incorrect thoughts in the reasonings of others, and to avoid them in our own.

Logic may therefore be defined as the science of the conditions on which correct thoughts depend, and the art of attaining to correct and avoiding incorrect thoughts.

Note.—This definition is in substance that given by Mr. Mill in his *Examination of Sir W. Hamilton's*

Philosophy, p. 391 (third ed., p. 448). 'Logic,' he says, 'is the art of thinking, which means of correct thinking, and the science of the conditions of correct thinking.' The word 'thoughts' is substituted for 'thinking,' in order to bring more prominently before the student, what Mr. Mill himself acknowledges, the fact that Logic is concerned with the *products* or results rather than with the *process* of thought, i. e. with *thoughts* rather than with *thinking*, though, in estimating the conditions on which correct thoughts depend, it is necessary, to some extent, to take account of the processes by which they are formed. It seems also desirable to introduce into the definition of Logic some reference to 'incorrect thoughts,' as bringing out more distinctly the character of Logic as an art, and asserting for it the right of investigating fallacies.

CHAPTER III.

On the Relation of Thought to Language.

WHETHER it is possible to think without the aid of language, is a question which has been a constant source of dispute amongst logicians and psychologists. It is not necessary, however, here to enter on this discussion. As all logicians are agreed that we cannot communicate our thoughts without the aid of language, or of equivalent signs, and that practically we do always think by means of language, by a sort of internal converse, it will be safer to adopt the terminology of those authors who regard our thoughts as expressed in language rather than that of those who consider or attempt to consider them in themselves as apart from their expression in words. I shall therefore speak of Terms and Propositions, not of Concepts and Judgments.

Note.—Sir W. Hamilton and his followers, regarding Logic as primarily and essentially concerned with thought, and only secondarily and accidentally with language, attempt to mark the products of thought by words which do not imply their expression in language. Thus, instead

of Terms and Propositions, they use respectively the words Concepts and Judgments. The word Syllogism, owing to the ambiguity of the Greek word λόγος, stands either for the internal thought or the external expression of it. (See Hamilton's *Lectures on Logic*, Lecture i.)

CHAPTER IV.

Division of the Products of Thought.

IT has been stated that thought is the act or operation of comparison. Its simplest result is that which is expressed by the Term. Terms may be combined into Propositions, and Propositions, either singly or in conjunction with one or more other propositions, may lead to Inferences. I shall treat in order of the Term, of the Proposition, of Inferences. Before proceeding further, it may perhaps be useful to the student to give by anticipation instances of these products or results of thought. Man, good, manliness, goodness, the goodness of man, the virtue of manliness, are all instances of terms. 'This man is good,' 'All citizens of a state are under an obligation to obey its laws,' are instances of Propositions, and, according to logical phraseology, 'good' is said to be *predicated* of 'this man,' and 'under an obligation to obey its laws' is said to be *predicated* of 'all citizens of a state.' The term predicated is called the *Predicate*, and the term of which it is predicated is called the *Subject*, the word 'is' or 'are' (or, in the case of negative propositions, 'is not' or 'are not'), which connects the two, being called the *Copula*. Lastly, we may take as instances of Inferences the following:—

No rectilineal figure is contained by less than three lines;
Therefore, no figure contained by less than three lines is a rectilineal figure.

All Englishmen are of mixed descent,
This is an Englishman;
Therefore, he is of mixed descent.

The last proposition (which is called the *conclusion*) is said to be *inferred* from the proposition or propositions which precede it (called the *premiss* or *premisses*).

PART I.—THE TERM.

CHAPTER I.

On the Various Kinds of Terms.

A TERM (so called from *terminus*, a boundary, because the terms are the two extremes or boundaries of the proposition) is a word or combination of words which may stand by itself as the subject or predicate of a Proposition; it expresses either an individual, a group of individuals, an attribute, or a group of attributes. This definition obviously excludes all articles, adverbs, interjections, conjunctions, prepositions, and oblique cases of nouns. It also excludes verbs; for though a verb expresses attributes, and often serves at once for the copula and the predicate (or part of the predicate), as in the propositions 'John walks,' 'William fears Thomas,' it must, in a logical proposition stated according to strict form, always be analysed into the copula and participle: thus the above propositions, when stated logically, become 'John *is* walking,' 'William *is* fearing Thomas[1].' A

[1] A proposition of which the verb is not analysed into the copula and a participle is called by the older logicians '*secundi adjacentis*,' in contradistinction to the form of proposition as ordinarily stated in Logic, which is called '*tertii adjacentis*.' Thus vir currit is a proposition *secundi adjacentis*, vir est currens is a proposition *tertii adjacentis*.

pronoun is only significant as standing in the place of a substantive, and therefore we may limit ourselves to the consideration of substantives, adjectives, and participles.

By the older logicians a term was defined as a *Categorematic* word (from κατηγόρημα 'something which can be predicated'), i.e. a word or combination of words which can stand by itself as the predicate of a proposition. All words or combinations of words which require to be joined with some other word or words in order to serve this purpose were called *Syncategorematic* words.

It has been said that a term expresses an individual, a group of individuals, an attribute, or a group of attributes. If it expresses an individual, as Socrates, the present Queen of England, the sea, hic, ille, hoc, illud, &c., it is called a (1) *Singular Term.* If it expresses a group of individuals, it may either be applicable to each individual of the group severally as well as to the group collectively, or to the group collectively but not to each individual severally. Thus the terms, man, horse, flower, are not only applicable to each individual of the groups which they express, but also to the groups collectively; I can say, 'John is a man,' 'Thomas is a man,' 'this is a horse,' 'that is a horse,' 'the rose is a flower,' 'the dahlia is a flower,' &c. But I cannot say 'Caius is the fourteenth legion,' 'Pompey is the Roman senate,' though I can predicate 'fourteenth legion' or 'the Roman senate' of the groups, taken collectively, which these terms express. In the former case the term is called a (2) *Common Term*, in the latter a (3) *Collective*

Term. A collective term may by a slight change of phraseology be expressed as a common term; thus, 'Roman senate' may become 'Roman Senators,' or 'fourteenth legion' 'soldiers of the fourteenth legion.' But, as it stands, a collective term is, in predication, as will be noticed hereafter, equivalent to a singular term.

A common term is equally applicable to each individual severally of the group which it expresses, and it is so in virtue of certain points of similarity which all the individuals possess in common. It is in fact because we have observed that they all possess certain attributes in common, that we are able to call them by a common name. Thus a common term, like man, horse, &c., at once suggests to me a certain group of individuals and a certain group of attributes which is predicable of each of these individuals severally. But there are other terms, called (4) *Attributives*, and (5) *Abstract Terms*, which express attributes or groups of attributes only. *Attributives* may be distinguished from Abstract Terms by the fact that they may form the predicate, but cannot, unless joined with a singular, collective, common, or abstract term, form the subject of a proposition. Grammatically, they are represented by adjectives and participles, when not used substantively. Of this class are the terms, human, red, heavy, brave, willing, thinking, running, &c., when not used substantively. Thus I may say, 'Socrates is human,' but, when I wish to employ the term 'human' in the subject of a proposition, I must append to it some such word as 'being,' and say 'this human being is

Socrates[2].' Such terms have, in fact, no signification, unless used substantively, or in conjunction with a substantive, either as qualifying it or as predicated of it.

Abstract Terms, on the other hand, not only express mere attributes or groups of attributes, but may be thought of without any reference to the individuals of which these attributes are predicable. Of this class are such terms as humanity, colour, figure, fortitude, &c. To it may be referred sentences employing the indicative mood and introduced by the word 'that,' infinitive moods, some instances of adjectives and participles used substantively, and generally every term, being neither singular, collective, nor common, which may be employed both as the subject and as the predicate of a proposition.

All the terms discussed above may be employed as predicates of propositions. Attributives alone cannot be employed as subjects.

We have thus divided terms into singular, collective, and common terms, attributives, and abstract terms. It must however be borne in mind that many of those attributives and abstract terms which are most frequently in use, or which have been for a long time in use, have come to be employed as common terms. Thus we sometimes speak of red, good, virtue, figure, number, pleasure,

[2] It is perhaps hardly necessary to remark that in such phrases as 'Great is Diana of the Ephesians,' 'Great is my rejoicing,' the places of the subject and predicate are reversed, for the sake of laying greater emphasis on the predicate. In a logical analysis of such propositions, the subject and predicate must be restored to their normal positions.

colour, &c. (both in the singular and plural number), exactly as if they were common terms, though they still retain in other connexions their use as abstract terms or attributives. This remark will be found of great importance in some of the subsequent sections.

Note 1.—Nothing has been said above of the common distinction between abstract and concrete terms. An abstract term would be defined as a term expressive of an attribute or group of attributes considered apart from the individuals of which it is predicable; a concrete term as a term expressive of an attribute or group of attributes considered in reference to the individuals of which it is predicable, as well as of an individual or group of individuals itself. The terms John, the tenth legion, man, human, would all be called concrete; humanity would be called an abstract term[3]. It will be noticed that I have availed myself of the expression 'abstract term,' but avoided, as too wide to be of practical service, the contrasted expression 'concrete term.' Concrete terms include what I have called attributives, as well as singular, collective, and common terms.

[3] Mr. Mill (*Logic*, bk. i. ch. ii. § 4) states that Locke and several later writers have applied the expression 'abstract name' to all general names, that is, attributives and common terms as well as what I have called abstract terms. This statement, however, is not uniformly true of Locke. See, for instance, Essay, bk. iii. ch. viii. Mr. Mill himself, following the practice of the Schoolmen, takes the expression in the same limited sense as in the text. By the older logicians singular and collective terms were not regarded as concrete, no account being taken of them in this distinction.

Note 2.—The term 'attributive' (which has already been employed by Harris and James Mill) is used in preference to the term 'adjective,' both because it includes participles, and because it seems undesirable in a work on Logic to employ a technical term of Grammar. Harris (*Hermes*, bk. i. ch. vi.) includes amongst 'attributives' verbs; but a verb, as has already been stated, is, in Logic, always represented by the copula and a participle.

I have placed attributives before abstract terms, because they are more nearly allied to singular, collective, and common terms, being, for the most part, either predicated of these terms or employed to qualify them. They seem also as a rule to precede abstract terms in their formation. Thus human, red, brave, good, willing, must have been employed before the corresponding terms humanity, redness, bravery, goodness, willingness.

Note 3.—For the sake of completeness, I have spoken of a term as expressing an attribute or a group of attributes. There is however no distinct name for a term expressing a single attribute incapable of analysis, and the only peculiarity of such terms is, as will be seen below, that they are incapable of definition. Locke called attributes which were incapable of analysis 'simple ideas,' but the expression 'simple term' would not be applicable in a corresponding sense.

Note 4.—That common terms, attributives, and abstract terms are formed from a comparison of individual objects or groups of objects, and that consequently they are results of thought, is obvious. But it may not be so

easy to perceive that this is the case with singular and collective terms. These terms however are appropriated to individual objects or groups of objects in order to distinguish them from others, and the necessity for such distinction can only arise after a comparison of this or that individual or group with others, and a perception of certain points of resemblance and difference between them. Unless I had observed some difference between John and Thomas, this table and that, the thirteenth legion and the fourteenth, it would never have occurred to me to distinguish them by separate names; but this very observation of a difference involves an act of comparison, and consequently an act of thought.

Note 5.—It is important to notice that in a series of terms, like man, human, humanity, all expressing the same attributes, the later and more abstract terms can hardly fail to suggest the earlier and more concrete, and it is so because the earlier terms of the series have been longer formed and are therefore, as a rule, more familiar to us. Thus 'humanity' can hardly fail to suggest to us the word 'human,' from which it is formed, and 'human' will suggest the word 'man,' from the Latin equivalent of which it is also formed, and whose attributes it expresses. Nor can we use the word 'man' without thinking of this or that individual man with whom we are familiar. A common term, in fact, expresses simply an individual object divested of all its peculiar attributes, and regarded as possessing only those attributes which it has in common with all the other objects which are desig-

nated by the same name. But it is indifferent on which object of the group the mind concentrates its attention, and we are all along conscious that the particular object selected is simply representative of the group. And hence it is that a common name simultaneously suggests to the mind a group of individual objects and a bundle of attributes characteristic of that group. For a further discussion of this subject, see Hamilton's *Lectures on Metaphysics*, Lect. xxxv. and xxxvi.; Mansel's *Prolegomena Logica*, ch. i.; and Mill's *Examination of Hamilton*, ch. xvii.

Note 6.—Mr. Mill maintains that attributives, when employed as predicates, are really common terms. Thus the propositions 'All triangles are three-sided,' 'All wise men are just,' are regarded by him as only abbreviated modes of saying 'All triangles are three-sided figures,' 'All wise men are just men.' I should allow that the attributive in the predicate, when taken in conjunction with the subject, always suggests a common term which may be substituted for it, as in the syllogism 'All wise men are virtuous, All virtuous men are happy; ∴ All wise men are happy.' But, though the attributive may always admit of being expressed as a common term, while it continues to be expressed as an attributive there seem to be present to the mind only attributes, whereas, when it becomes a common term, there seems also to be present a group of individuals possessing those attributes.

CHAPTER II.

On the Denotation and Connotation of Terms.

A TERM may be said to *denote* or designate individuals or groups of individuals, to *connote* or mean attributes or groups of attributes [1].

[1] It ought, perhaps, to have been stated in the earlier editions of this work that the term *connotation* is here employed in a somewhat different sense from that which is attached to it either in the scholastic logic or in the system of Mr. Mill.

In the scholastic logic, a *connotative* term is 'one which primarily signifies an attribute, secondarily a subject,' as 'white,' the contrasted term being called an 'absolute term,' as 'man' or 'whiteness.' See Mansel's Aldrich, cap. 1, § 3, note g.

According to Mr. Mill's nomenclature, a connotative term is one which 'denotes a subject and implies an attribute.'

By Mr. Mill, not only singular and collective, but also abstract terms are regarded as non-connotative. In the scholastic logic, what I have called attributives are alone recognised as connotative terms. See Mill's Logic, Bk. I. ch. ii. § 5.

As the term is already employed with so much uncertainty, it appears to me not inexcusable to claim still further licence, and to appropriate the expressions 'denotation' and 'connotation' of 'terms' in a sense parallel to that which is expressed by the distinction between the 'extension' and the 'intension' of a 'notion' or 'concept,' applying 'denotation' simply to the objects, and 'connotation' simply to the attributes which are signified by a term. This is a broad and exceedingly convenient distinction, and, notwithstanding the apparent paradox involved in it, namely, that 'abstract terms are not denotative,' I believe that the general employment of the expressions in this sense would considerably simplify the statement and explanation of many logical difficulties.

In the first place, a term may serve to *denote* or point out an individual object or group of individuals. Thus 'Socrates' denotes or points out and distinguishes from all others the individual man Socrates. The expression 'tenth legion' denotes or points out, and distinguishes from all other collections of men, the particular group known as the tenth legion. Similarly, the word 'man' denotes or points out, and distinguishes from all other groups, a certain group of individuals to each member of which and to each member of which only the word 'man' may legitimately be applied. All terms of this kind, therefore, viz. singular, collective, and common terms, are denotative; but terms like human, white, humanity, whiteness, i.e. attributes and abstract terms, are not denotative, except mediately, that is, so far as they suggest the common terms 'human beings,' 'white things.'

In the second place, a term may serve to *connote* attributes or groups of attributes. Thus terms like humanity, human, man, viz. abstract, attributive, and common terms, are all connotative, that is, they at once suggest or imply attributes. But singular and collective terms like 'Socrates,' 'the tenth legion,' are not connotative, except so far as they suggest common terms. This remark requires some explanation. A collective term like 'the tenth legion,' 'the House of Commons,' at once suggests the corresponding common term, 'soldiers of the tenth legion,' or 'members of the House of Commons'; and this common term may

connote any number of attributes, but, as the attributes are suggested mediately through the common term and not directly by the collective term, the collective term is, strictly speaking, non-connotative. The same is the case with a singular term. A term like 'William' may suggest to me 'man,' 'male,' 'Englishman,' 'one of my friends,' &c., and so may become connotative, but it is in itself rightly regarded as non-connotative, inasmuch as it suggests to me these attributes only through the medium of the common terms to which it is referred.

It appears therefore that common terms are both denotative and connotative; that singular and collective terms are denotative, but not connotative; that abstract terms and attributives are connotative, but not denotative; and finally, that mediately, as suggesting common terms, any non-connotative term may become connotative and any non-denotative term denotative.

Note.—The distinction between the denotation and connotation of a term is often otherwise expressed, as that between the *extension* and *intension* (or *comprehension*), or the extensive and intensive (or comprehensive) capacity, or the *breadth* and *depth*, of a notion. Having adopted the phraseology which designates the simplest product of thought as a term, instead of a notion, I shall speak of the extensive and intensive (or comprehensive) capacity of a term. The extensive capacity of a term is measured by the number of individuals which it

designates (denotes), the intensive or comprehensive capacity of a term by the number of attributes which it includes, suggests or implies (connotes). It is plain that in a series of common terms, standing to one another in a relation of subordination, the denotation and connotation, or the extensive and intensive capacities, of the term are so related, that as the one increases the other decreases, and vice versâ. Thus, if we arrange in order any series of common terms, as flower, rose, moss-rose, we see that 'flower,' which implies the smallest number of attributes, is applicable to the largest number of individuals; 'moss-rose,' which is applicable to the smallest number of individuals, implies the largest number of attributes: and generally in any series of common terms arranged in subordination, the larger the denotation or extensive capacity, the smaller is the connotation or intensive capacity, and vice versâ. In conformity with this principle, the singular term which stands for the individual, and is smallest in denotation, is, when we refer it to the various common terms which may be predicated of it, and so assign to it mediately an intensive capacity, the largest in connotation. Thus the term Socrates, when I regard it as expressing one who was a philosopher, a teacher, a martyr, a soldier, an Athenian citizen, &c., &c., suggests to me far more attributes than any one of these common terms singly.

PART II.—THE PROPOSITION.

CHAPTER I.

On the Subject and Predicate.

A PROPOSITION asserts or denies, as the result of comparison, some word or combination of words of some other word or combination of words, as e. g. 'James is the man I saw yesterday'; 'No rectilineal figure is contained by less than three lines'; 'Some stars are not planets.' As before stated, the words or combinations of words thus employed are called *terms*, the term affirmed or denied is called the *predicate*, the term of which it is affirmed or denied the *subject*, the connecting verb, whether qualified or not by the negative particle, the *copula*, and the predicate is said to be *predicated* of the subject. In the above examples, 'the man I saw yesterday,' 'contained by less than three straight lines,' and 'planets' are *predicates* and are *predicated* respectively of 'James,' 'all rectilineal figures,' and 'some stars,' as *subjects*. In the first case the predicate is predicated affirmatively, a fact which is expressed by the copula 'is'; in the two last negatively, a fact which is

expressed by the copula 'is not.' These remarks may appear inconsistent with the form of the second example, but 'no rectilineal figure is, &c.' is really an abbreviated and unambiguous mode of stating the longer and ambiguous proposition 'All rectilineal figures are not, &c.'

The word 'predicated,' as equivalent to 'asserted or denied,' is here used in a wider than its ordinary signification. In common language, we say such and such an attribute cannot be predicated of such and such a term, using 'predicated' as equivalent to 'asserted' and as opposed to 'denied.' All ambiguity may be avoided by speaking of the predicate as predicated affirmatively or predicated negatively of the subject.

CHAPTER II.

On the Copula.

THE Logical Copula, it being its office simply to serve as a sign of predication, is limited to the present tense of the verb 'to be,' with or without the addition of the negative particle, according as the proposition is negative or affirmative. This limitation follows from the fact that it is simply the office of the proposition to express my present judgment as to the compatibility or incompatibility of two terms. Hence all reference to time, past or future, and even to time present, as respects the terms themselves, and not my judgment as to their compatibility, must be expressed in the predicate and not in the copula. I may, for brevity's sake, say 'fire burns,' 'Alexander was the son of Philip,' 'The guns will be fired to-morrow,' and, in conversation or discussion, it would undoubtedly be pedantic to express the propositions otherwise; but formally, for the purpose of being estimated logically, I must resolve them into their logical elements, and say 'Fire *is* burning,' 'Alexander *is* a person who was son of Philip,' 'The firing of the guns *is* an event which will take place to-morrow.'

Not only does the logical copula convey no notion of time with reference to the terms themselves (or, to speak more accurately, the things signified by them), but it is also divested of the notion of existence. In other words,

it is employed simply as a connecting particle, not as a substantive verb. Where the substantive verb is used in a logical proposition, it must be expressed in the predicate. Thus 'I am,' 'The king is not,' become 'I *am* existent,' 'The king *is* non-existent.' That the copula implies no notion of existence is evident from the fact that we can use such propositions as these: 'The labours of Hercules are a myth,' 'He is a nonentity.'

Can we modify the copula so as to express certainty, probability, possibility, or other modes of connexion between the subject and the predicate? This is the celebrated question of Modality, a question which has been the source of much difference of opinion amongst logicians. Even though it were granted that the proposition simply expresses our present judgment on the compatibility or incompatibility of two terms, it might be maintained that it should express the nature of our judgment and the degree of our assent or dissent, whether it be certain, approximating to certainty, or faltering. Thus it might be maintained that the following should be accepted as instances of the ultimate analysis of a logical proposition: 'This *is certainly* the man I saw yesterday,' 'This *is probably* the man I saw yesterday,' 'This *is possibly* the man I saw yesterday.' That we use these forms in conversation and discussion is unquestionable, but it is one main object of Logic to analyse our abbreviated inferences and statements into their full logical equivalents. Instead therefore of admitting various descriptions of copulæ (other than the affirmative and negative), in order to adapt

Logic to ordinary language, it seems simpler, as well as more scientific, to insist on the uniform character of the copula, and to represent propositions like the foregoing as predicating our degree of assent to or dissent from the sentence in question. Thus, after asking myself the question 'Is this the man I saw yesterday?' I may either answer simply 'This *is* the man I saw yesterday,' or I may describe the degree of my assent by stating 'That this is the man I saw yesterday *is* certain, probable, possible,' &c. In fact, such propositions seem to be the result of an act of reflexion on the degree of our own conviction.

I shall therefore regard the form A *is* or *is not* B as the ultimate and uniform logical analysis of all propositions, though I shall occasionally, for the sake of brevity, avail myself of the forms sanctioned by popular language.

Note.—As regards the expression of time in the copula, the student will find the opposite opinion to that taken in the text adopted by Mr. Mill, *Logic*, vol. i. ch. iv. § 2. In support of my view, he may refer to Dr. Mansel's *Prolegomena Logica*, pp. 63, 64. On the question of expressing in the copula the degrees of assurance with which a proposition is entertained ('certainly,' 'probably,' &c.), see Sir W. Hamilton's *Discussions*, pp. 145–7, and, for a more qualified view than that taken either by Sir W. Hamilton or myself, Dr. Mansel's *Prolegomena Logica*, note G.

CHAPTER III.

Division of Propositions according to their Quantity and Quality.

WE have already seen that propositions are either *affirmative* or *negative*, according as the copula used is of the form 'is' or 'is not.' This is called a division of propositions according to their *Quality*.

They are further divided, according to their *Quantity*, into *Universal* and *Particular*. For, in affirming or denying a predicate of a subject, it is obvious that I may either affirm or deny the predicate of all the individuals denoted by the subject, or of part only. Thus in affirming mortality of man, I may say 'All men are mortal,' or 'Some men are mortal'; in denying wisdom of man, I may deny it of all men or only of some men, i.e. I may say 'No men are wise,' or 'Some men are not wise.' When the predicate is affirmed or denied of all the individuals denoted by the subject, the proposition is called an Universal Proposition; when of part only, a Particular Proposition. A Singular Proposition, i.e. a proposition of which the subject is a singular term, ranks as an Universal, because the predicate is affirmed or denied of everything (i.e. in this

case, the one thing) denoted by the subject. The same remark holds good of a proposition in which the subject is a collective term. An attributive, as we have already seen, cannot, by itself, be used as the subject of a proposition. Abstract terms which have come to be used as common terms, and admit of plurals, as figure, triangle, virtue, pleasure, &c., have a denotative power, and may, like common terms, form the subjects of either universal or particular propositions. But those abstract terms, like humanity, wisdom, &c., which retain their original characteristic of being connotative only, and admit of no plurals, simply express an attribute or group of attributes with which, as a whole, it is asserted or denied that the predicate is compatible; consequently, a proposition, of which such a term is the subject, ranks as an universal.

Thus such propositions as 'Ambition is aggressive,' 'Wisdom is a virtue,' 'The fourteenth legion is disbanded,' 'Socrates is an Athenian citizen,' are, on the very face of them, universals. But propositions in which the subject is a common term or an abstract term used as a common term, must be *quantified*; that is, we must attach to the subject either an universal or a particular designation. It is not sufficient to say, 'triangles are figures,' 'horses are black'; we must state whether we mean that 'all triangles' or 'some triangles' are 'figures,' whether we mean that 'all horses' or 'some horses' are black. 'Indefinite' or 'indesignate' propositions, as they are called, i.e. propositions in which the subject, being

a common term, is not quantified, are inadmissible in Logic.

By combining the division of propositions into universal and particular with that into affirmative and negative we obtain four forms, viz.—

Universal Affirmative.	All X is Y.	(A)
Universal Negative.	No X is Y.	(E)
Particular Affirmative.	Some X is Y.	(I)
Particular Negative.	Some X is not Y.	(O)

I shall in future designate these forms of proposition respectively as A, E, I, O[1].

Note.—Sir W. Hamilton[2], followed by several other logicians, maintains that *in thought* the predicate is always quantified as well as the subject. He proposes to reform the logical theory of the proposition accord-

[1] It sometimes requires a little ingenuity to state a given proposition in one of the above forms. Thus the propositions 'None but the brave deserve the fair,' 'The wise alone are good,' 'Not every historian is worthy of credit,' 'All his acts are not defensible,' when stated in strictly logical form, become respectively, No not-brave (or None who are not brave) are deserving of the fair, No not-wise (or None who are not wise) are good, Some historians *are not* worthy of credit, Some of his acts *are not* defensible. The simplest equivalents of the two former propositions are, All who deserve the fair are brave, All good men are wise; but these are arrived at by permutation and conversion, two forms of inference which have not yet been explained.

[2] Sir William Hamilton's theory was anticipated in a work now little read, but full of original suggestions on logical questions, Mr. George Bentham's Outline of a New System of Logic, published in 1827.

ingly, and in lieu of the four ordinary forms of proposition substitutes the following:—

All X is all Y.
All X is some Y.
All X is not any Y.
All X is not some Y.

Some X is all Y.
Some X is some Y.
Some X is not any Y.
Some X is not some Y.

This scheme, if adopted, would, as Sir W. Hamilton shews, reduce all conversion to simple conversion, render nugatory any discussion as to the distribution of terms, and considerably simplify the forms of syllogism: see the Appendices to Sir W. Hamilton's *Discussions*, and to his *Lectures on Logic*. Amongst other criticisms may be seen Mr. Mill's in his *Examination of Hamilton's Philosophy*, ch. xxii. It would of course be undesirable to enter here into any discussion as to the merits of Sir W. Hamilton's theory, but, as reasons for not adopting the quantification of the predicate in the present work, it may be sufficient to state (1) that, as to utility, the trouble entailed by quantifying the predicate in every proposition would probably far exceed that saved by simplifying the forms of Conversion and Syllogism; (2) that the forms of expression given above are not merely unusual, but are such as we never do use; whereas, though the analysis of our thoughts frequently leads to

forms of expression which are unusual, this would, if admitted, be the only case in which it led to forms which are never used at all; (3) that some of the above propositions really contain in a compressed form two ordinary propositions, as e. g. 'All A is all B,' contains the two ordinary propositions 'All A is B' and 'All B is A,' the proposition 'Some A is all B' contains the two ordinary propositions 'Some A is B' and 'All B is A,' whereas it is the object of Logic not to state our thoughts in a condensed form but to analyse them into their simplest elements.

CHAPTER IV.

Distribution of Terms.

A TERM is said to be *distributed*, when it is employed in its entire extent, i. e. when it applies to all the individuals denoted by the name. Thus, when we say, 'all men,' 'no men,' 'man' is distributed; when we say 'some men,' it is undistributed. This phraseology of course applies directly only to common terms; but singular and collective terms, as has already been explained, are always taken in an universal acceptation, or, in other words, are always distributed. The same is true of those abstract terms which have not come to be used as common terms, because they, as it were, personify the attribute or group of attributes which they express; so far as regards distribution, they are, in fact, virtually singular terms. In such propositions as, 'This is wisdom,' 'Wisdom is justified of her children,' 'Warmth is essential to growth,' 'Knowledge is power,' it is obvious that the abstract terms are distributed precisely as if they were singular terms, and that, for all logical purposes, these propositions rank as universal. Attributives, i.e. adjectives and participles, have no meaning except in connexion with a substantive. They must either be prefixed to a substantive or predicated of it: we cannot say 'human' alone; we must either speak of 'human being,' 'human nature,' &c.,

or say, 'So and so is human.' Consequently the distribution or non-distribution of an attributive, as 'human,' 'red,' &c., follows that of the corresponding common term, 'human being,' 'red thing,' &c.

Hence we perceive that a singular, collective, or abstract term is distributed wherever it may occur in a proposition. We have therefore only to enquire as to the distribution of common terms and attributives. With regard to these, the two following rules may be laid down.

1. All universal propositions distribute their subject, whereas particular propositions do not. This rule is obvious. The very word 'all' or 'no' shews that the subject is distributed, whereas the word 'some' shews that it is undistributed.

2. All negative propositions distribute their predicate, whereas affirmative propositions do not. For in every negative proposition we necessarily exclude from the subject every individual denoted by the predicate, but in an affirmative proposition we do not necessarily include in the subject every individual denoted by the predicate. Thus, if I say, 'No crows are yellow,' 'Some cherries are not red,' I exclude from the group 'crows' every individual object denoted by the term 'yellow things,' and from the group 'some cherries' every individual object denoted by the term 'red things[1].' But if I say, 'All

[1] Here it will be noticed that the terms 'yellow' and 'red,' though not in themselves denotative, suggest the corresponding common terms, 'yellow things' and 'red things,' and thus, through the medium of the common terms, become denotative.

men are animals,' or 'Some Englishmen are poets,' I do not include in the group 'men' all animals, nor in the group 'Englishmen' all poets. It may of course happen that the predicate in an A or I proposition is co-extensive with the subject, as in the propositions 'All men are rational animals,' 'Some men are poets,' but this fact is accidental, and is not implied in the form of the proposition.

From these two rules we infer that, in the case of common terms and attributives, an A proposition distributes its subject only, E both its subject and predicate, I neither, O its predicate only. If a term be singular, collective, or abstract, it is invariably distributed.

CHAPTER V.

Relation of the Predicate to the Subject of a Proposition (Heads of Predicables.)

N.B. *For this Chapter, in the case of beginners, may be substituted Appendix, p.* 160. *See Preface, p.* x.

FROM what has already been said, it is plain that a singular, collective, or abstract term, inasmuch as it is always distributed, cannot form the subject of an I or O proposition: on a little reflexion it will also be plain that the predicate of a proposition cannot be singular, collective, or abstract, unless the subject be the same. I have already noticed that an attributive can never form the subject of any proposition. These considerations will be found to simplify the problem before us.

This problem may be stated thus: How may the predicates of propositions be classified in relation to their subjects? or What are the heads of predicables (prædicabilia, things or words that may be predicated)? I shall discuss the four forms of proposition in order.

To commence with A, and with the special case where both subject and predicate are common terms, or abstract

terms which are used as common terms[1]. Here the predicate may either be (1) equivalent in extent to the subject, or (2) greater; it cannot be less. We can say, for instance, 'All men are animals,' or 'All men are rational animals,' but we cannot say 'All animals are men.' Now, if the predicate be (1) equivalent in extent to the subject, it is either (α) a Synonym, as 'A wold is a down'; or (β) a Definition, as 'A plane triangle is a three-sided rectilineal figure'; or (γ) a combination of a *genus* (a term which will be explained immediately) with some attribute which is peculiar to the term in question (called by Aristotle an ἴδιον or 'peculiarity'), as 'A plane triangle is a rectilineal figure the sum of whose angles is equal to two right angles.' A synonym, it need hardly be stated, is an equivalent word, and a definition is an exposition of the connotation of a term. Now it will be observed that the definition of a plane triangle consists of two parts, one in which it is designated as a 'rectilineal figure'—a wider group, which includes not only triangles but other rectilineal figures—and the other part an attribu-

[1] It may perhaps assist the student in following what, to be exhaustive, must necessarily be a tedious disquisition, if I afford him some clue to the order in which the cases will be discussed :—

In the A proposition, (1) if the predicate be a common term, the subject may be (α) a common, (β) a singular or collective term; (2) if the predicate be a singular or collective term, the subject must be the same; (3) if the predicate be an abstract term, the subject must be the same; and (4) if the predicate be an attributive, the subject may be (α) a common, (β) a singular or collective, (γ) an abstract term. The discussion of the cases in I, E, and O is much less elaborate.

tive, 'three-sided,' which distinguishes triangles from all other groups contained in the wider group to which triangle has been referred. The former part of the definition is called the *genus*, and with reference to it 'plane triangles,' the group of figures defined, are called *a species*; the latter part of the definition, 'three-sided,' which distinguishes triangles from squares, pentagons, and other rectilineal figures which are designated by the wider term, is called the *differentia* or 'differencing' attribute. It is obvious that there might be more than one of these, and then they would be called the *differentiæ*. With reference to the third case, 'A plane triangle is a rectilineal figure the sum of whose angles is equal to two right angles,' 'rectilineal figure' is, as before, to be regarded as a genus, but the attributive 'having the sum of its angles equal to two right angles' cannot be regarded as a differentia, for it is not connoted by the term 'plane triangle,' but requires to be proved of it; at the same time it is an attribute *peculiar* to the plane triangle, and hence, retaining the Aristotelian term, we may call it an ἴδιον. Lastly, there remains (2) the case in which the predicate is of greater extent than the subject, both being common terms, as in the propositions 'All men are animals,' 'All triangles are figures.' Here the subject denotes a smaller group of individuals included under the wider group designated by the predicate; that is, according to the nomenclature already explained, the predicate is related to the subject as *genus* to *species*. This relation is often spoken of as that of the *containing* to the *contained*.

When the predicate is a common term, and the subject a singular or collective term, as in the instances 'Socrates is a philosopher,' 'Socrates is an Athenian citizen,' 'The House of Commons is a branch of the legislature,' the predicate is related to the subject as a group of individuals to an individual, i. e. as a *species* to an individual, for the word 'genus' is only applicable to a group containing other groups. The same account must be given of those propositions in which the predicate is an abstract term employed as a common term, and the subject is an abstract term employed strictly as such, as e. g. 'Temperance is a virtue,' 'Heat is a mode of motion.'

When the predicate is a singular or collective term, the subject must, as we have already seen, be a similar term. Moreover the predicate in these propositions is always distributed as well as the subject, and consequently the two terms are co-extensive. But, inasmuch as singular and collective terms have, at least directly, no connotation, the predicate cannot stand to the subject in the relation of a definition, or of an ἴδιον[2] (i. e. an attribute peculiar to the subject) combined with a genus. It can only be (1) a synonym, or (2) a singular or collective term designating the individual or the collective group. As instances of these propositions we may take the following: 'Cephas is Peter,' 'Socrates is the son of Sophroniscus,' 'This is the man whom I saw yesterday, and whom I told

[2] The English word 'characteristic' might perhaps be employed as the equivalent of the Greek ἴδιον.

to come to me,' 'The fourteenth legion is the legion quartered in Britain.' When the predicate is not a synonym, it may perhaps be called a Designation.

When the predicate is an abstract term, the subject must be abstract as well, and, as in the case of singular and collective terms, the subject and predicate are both distributed, and consequently are of equal extent. We may take as instances of the various forms which propositions of this kind assume, 'Charity is love,' 'Honesty is the best policy,' 'Definition is the exposition of the connotation of a term.' Now in the first example 'love' is intended as a synonym of 'charity,' in the second 'the best policy' is predicated of 'honesty' as distinguishing it from all other courses of conduct; the third example is an ordinary case of a definition. The case of two abstract terms may therefore be regarded as identical, so far as concerns the relation of the predicate to the subject, with that of two common terms which are co-extensive.

Lastly, there remains the case in which the predicate is an attributive. Supposing the subject to be a common term, the predicate may, as we have seen, be (1) a *differentia*, as in the proposition 'All triangles are three-sided,' or (2) an ἴδιον, as in the proposition 'All plane triangles have the sum of their angles equal to two right angles.' Or, though neither a differentia nor an ἴδιον, it may be (3) what, along with the ἴδιον, modern logicians would call a *Property*, viz. an attribute which, though not connoted by the subject, nor even peculiar to it, follows from something connoted by the subject, either (α) as effect from cause or (β) as

conclusion from premiss[3]. An instance of (*α*) a property which follows from the connotation of the subject *by demonstration* would be given in the proposition 'A parallelogram has its opposite sides equal,' or in the proposition 'A circle is a figure the ratio of whose circumference to its diameter is approximately 3.14159 : 1.' As instances of propositions in which the predicate expresses (*β*) a property following from the connotation of the subject *by causation*, we may take 'Men are capable of combining for purposes of joint action,' 'Water communicates pressure equally in all directions.' The first property follows from rationality, or that combined with the power of articulate speech, which seems to be connoted by the very word 'man'; the second property follows from the fluidity of water. The last example of a property following by way of demonstration and the first of a property following by way of causation are ἴδια in the sense of Aristotle, as well as propria in the sense of the later logicians. There is (4) one other case, that in which the attributive, though neither connoted by the subject nor following from anything connoted by the subject, is predicable of everything denoted by it. Such an attributive is called an *Inseparable Accident*, and the common instance given by logicians is the proposition 'All crows are black.' If the blackness of crows could be connected by way of causation with any

[3] The student must bear in mind that every ἴδιον is a *Property*, though every *Property* is not an ἴδιον. The definition given above applies only to those properties which are not ἴδια. The formal definition of *Property* will be found on p. 45.

attribute connoted by the name, it would be regarded as a property; if, on the other hand, a crow could be found which was not black, blackness would be degraded to the rank of a Separable Accident, a term which will be explained below.

We have thus far considered attributives as predicates in the special case where the subject is a common term. Where the subject is a singular or collective term, and consequently has no direct connotation, the attributive forming the predicate can only be an inseparable accident[1]. I may indeed say 'Socrates is rational,' but I predicate rationality of him as man, not as Socrates. Where the subject is an abstract term, the attributive which forms the predicate may be a differentia or a property, but cannot be an inseparable accident, for an abstract term denotes no individuals.

In the I proposition we are not concerned with singular, collective, or abstract terms. If the predicate be (1) a common term, it may be related to the common term in the

[1] Sometimes a distinction is drawn between the separable and inseparable accidents of an individual. An inseparable accident of an individual is regarded as one which is predicable of it at all times, a separable accident as predicable only at certain times. Thus in the propositions, 'John is tall,' 'John is sitting down,' 'tall' is regarded as an inseparable, 'sitting down' as a separable accident. But the expressions separable and inseparable accident are here employed in an entirely different sense from that in which they have been employed above, and it seems preferable to regard all attributes which are predicated of individuals, in their individual character, as inseparable accidents—inseparable, that is to say, from the individual under the circumstances in which it is at present placed.

subject either (α) as *species* to genus, or (β) as species to species, as e. g. 'Some men are poets,' or 'Some poets are philosophers.' The latter relation is that of two groups, which have some members in common—*overlapping species*, as they have been called. The relation may also be (γ) that of *genus* to species, as 'Some men are animals,' 'Some poets are men,' but this form of proposition is practically useless, as it stops at a particular assertion when an universal assertion is legitimate. If the predicate be (2) an attributive, it must, unless the proposition state less than the truth, be a *Separable Accident*, i. e. an attribute which is true only of some of the individuals denoted by the subject, as e. g. 'Some men are black,' 'Some triangles are equilateral.'

With regard to negative propositions, it is not necessary to speak at any length. The negation of an attributive in an E proposition implies that it is neither a differentia, property, nor accident; in an O proposition that it is neither a differentia, property, nor inseparable accident, though it may be a separable accident. All that has been said of the relations of singular, collective, and abstract terms, as subjects, either to similar terms or to common terms and attributives, as predicates, in the case of an A proposition, holds good also, *mutatis mutandis*, of similar relations in E. It may perhaps be as well to give a few instances of such propositions: 'Socrates *is not* a poet,' 'Socrates *is not* the man I saw yesterday,' 'The fourteenth legion *is not* engaged in the battle,' 'Charity *is not* obtru-

sive,' 'Ambition *is not* a virtue.' Where a common term is denied of a singular or collective term, it stands to the subject in the relation of species to individual. Where a common term is denied of a common term, there is nothing to prevent the terms being extremely remote from each other and hardly admitting of comparison; but here the most appropriate relation is—in an E proposition, that of co-ordinate though exclusive species, i.e. of species which having many characteristics in common, and both falling immediately under the same genus, still denote no individuals in common,—in an O proposition, that of overlapping species. Thus we should be far more likely to derive information from such propositions as these, 'No sandstone is limestone,' 'Some astronomers are not mathematicians,' than from such propositions as these, 'No men are trees,' 'Some stones are not vipers.'

From what has been said we have derived the following names for the predicate in its relation to the subject: (1) *synonym*, (2) *definition*, (3) *designation*, (4) *genus*, (5) *species*, (6) *differentia*, (7) ἴδιον, (8) *property* (*not being an* ἴδιον), (9) *inseparable accident*, (10) *separable accident*. Of these, logicians have neglected synonym and designation, the former probably on account of its slight importance, the latter perhaps because it applies only to singular and collective terms. Definition is analysed into genus and differentia; no distinction is drawn by later logicians between ἴδια and those properties which are not peculiar to the subject, both being alike designated by the word 'property';

and the word 'accident' serves alike for separable and inseparable accident. Genus, difference, species, property, and accident, are known as the *five heads of predicables.*

These may be briefly defined as follows:—

A *Genus* is a common term expressive of a wider group of individuals including narrower groups.

A *Species*, in reference to a genus, is a common term expressive of a narrower group included in the genus; in reference to an individual, of a group including it.

A *Differentia* is an attributive which expresses part of the connotation of some common term, and which distinguishes that term from all other species which fall under the same genus.

A *Property* is an attributive which does not express any part of the connotation of the common term, but which follows from some part of the connotation of the term, either as an effect from a cause, or as a conclusion from premisses.

An *Accident* is an attributive which may be predicated of the whole or part of the individuals denoted by a common term, or which may be predicated of an individual, but which is neither connoted by the common term nor to be inferred from anything which is connoted thereby [5].

[5] In the above definitions, wherever the expression 'common term' occurs, it must be understood as including those abstract terms which have come to be employed as common terms.

Note 1.—Contrary to the practice of most logicians, I have discussed the heads of predicables under the second part of Logic. In taking this course, I have attempted to restore to them their original significance in the works of Aristotle, as a classification of the predicates in their relation to the subjects of propositions. It is perhaps needless to add, that I have not followed implicitly the Aristotelian account, or in all cases adopted the Aristotelian phraseology. By Porphyry, the schoolmen, and their successors of the seventeenth and eighteenth centuries, the heads of predicables were regarded as a classification of universals in their relations to one another, rather than with reference to their place in a proposition, and consequently, from the time that Logic was divided into parts, were discussed under the first part. Definition and division, as dependent on a knowledge of genus, species, and differentia, are also here treated under the second part of Logic, though they are ordinarily discussed under the first.

Note 2.—On the vexed and subtle question of the Import of Propositions, or, as it is sometimes called, the Theory of Predication, the student may be referred to Hobbes' *Computation or Logic*, ch. iii., Mr. Mill's *Logic*, Bk. I. ch. v., Dr. Mansel's *Prolegomena Logica*, ch. ii., Sir W. Hamilton's *Lectures on Logic*, Lects. viii. and xiii., and Mr. Mill's *Examination of Sir W. Hamilton's Philosophy*, ch. xviii. The view I should adopt may be briefly stated as follows: wherever the predicate is a singular or collective term, or, though a common or

abstract term, a synonym of the subject, the theory of Hobbes, that the predicate is a name of the same things of which the subject is a name, furnishes a sufficient account; in all other cases, Hobbes' theory is true, though insufficient, for, where the predicate is an attributive or an abstract term (not being a synonym), the predicate also asserts or denies certain attributes of the subject, and where it is a common term (not being a synonym), not only are certain attributes asserted or denied of the subject, but the latter is referred to or excluded from the group of individuals denoted by the predicate. Thus, in the proposition 'Socrates is the son of Sophroniscus,' the predicate may be regarded as simply another name for the subject; but when we say 'Socrates is human,' we imply more than this, namely, that certain attributes connoted by the word 'human' may be affirmed of Socrates. Further, when we say 'Socrates is a man,' we not only assign another name to the subject, and predicate of it certain attributes, but we also include it in a certain group. This peculiarity follows from the double signification of common terms, by which they both denote a group of individuals and connote a group of attributes, or, in other words, simultaneously suggest to the mind a group of individual objects and a bundle of attributes characteristic of that group. From what has been said it will be seen that I do not agree with Mr. Mill in regarding all predication, other than that of a singular or collective term, as a predication merely of attributes.

CHAPTER VI.

On Verbal and Real Propositions.

AFFIRMATIVE propositions in which the subject is a common or abstract term may be divided into Verbal and Real. A verbal proposition expresses merely the connotation or part of the connotation of the term, a real proposition expresses either solely, or in conjunction with part of the connotation of the term, properties, accidents, or both. Thus a verbal proposition simply states what might be gathered from a due consideration of the name, as 'All men are rational,' 'All triangles are three-sided'; whereas a real proposition imparts knowledge which could not be gathered from the name alone, as 'All triangles have the sum of their angles equal to two right angles,' 'Some men are black.' It is plain that all particular affirmative propositions are real.

Note.—The distinction between verbal and real propositions is otherwise expressed by that between Analytical and Synthetical judgments (Kant), Explicative and Ampliative judgments (Sir W. Hamilton and Abp. Thomson), Essential and Accidental propositions (the schoolmen). Tautologous propositions (All A is A), and propositions in which the predicate is a synonym of the subject (as e. g. 'Charity is love'), should be referred to the head of verbal propositions.

CHAPTER VII.

On Definitions.

A DEFINITION is an exposition of the connotation of a term. It may always be represented in the form of a proposition, of which the term defined forms the subject and the exposition the predicate[1].

As a definition expounds or enumerates the attributes which a term implies, it is plain that singular and collective terms, inasmuch as they do not in themselves connote any attributes, are incapable of definition. They may, however, be *described* by means of the various common terms which are predicable of them, as well as by designations which are peculiar to themselves. Thus I may say 'John is a tall man of fair complexion, is by profession a London solicitor, and occupies such and such a house in Bedford-row.' This may be called a *Description of a Singular or Collective Term*, in order to distinguish it from the Description of a common term, which is noticed below, and which I shall call simply Description. Besides singular and collective terms, a term expressing a single attribute, which is incapable of analysis into other attributes, is incapable of definition. Thus it has been maintained that it is useless to attempt to define such terms as pleasure, pain, colour, thing, attribute, &c.

[1] It is indifferent whether we speak of the entire proposition as the definition, or merely the exposition which forms the predicate; similarly in the case of a description.

In defining a term, it would of course be impossible, in every case, to state the definition in terms expressive of attributes which are themselves incapable of analysis, even if we were agreed as to what are terms of this character. The terms employed in the definition may therefore express groups of attributes, provided that, when taken together, they exhaust the connotation of the term defined. Thus each of the terms 'under one supreme head' and 'political government' expresses a large group of attributes, but if, when taken together, they exhaust the connotation of the word 'monarchy,' the definition may be accepted as legitimate.

In seeking to define a term, I invariably contrast it with some other term; often, when I imagine it to be difficult to exhaust the connotation, with a variety of other terms, from which I seek to distinguish it. Thus in seeking to define 'monarchy,' I contrast it with other forms of government; in seeking to define 'plane triangle,' I contrast it not only with other rectilineal figures but with spherical triangles; in seeking to define 'light,' I contrast it with heat, sound, electricity, and other impalpable powers of nature. Now that portion of the definition which is common to the term defined and to the other terms with which it has been compared is called the *genus*, the term defined standing to it in the relation of a species, as has already been explained; that portion which distinguishes the term defined from the terms with which it has been compared is called the *differentia*, or sometimes, when there is more than one distinguishing

attribute, the *differentiæ*[2]. Thus, if we were distinguishing rectilineal triangle from spherical triangle, 'three-sided figure' would be the genus; if from other rectilineal figures, the genus would be 'rectilineal figure'; if from both spherical triangles and other rectilineal figures, the genus would be simply 'figure.' It will be observed that the genus is always expressed in the form of a common term, or of an abstract term which is used as a common term, and that it may be qualified by an attributive. This is the case even in defining abstract terms, as e. g. in the definitions, 'Justice is a virtue which respects our relations to other men in society,' 'Temperance is a virtue which respects the control of our own desires,' the genus[3] is 'a virtue,' an abstract term which has come to be used as a common term.

A Definition being an exposition of the meaning of a term, it is obvious that all definitions must be limited by the state of our knowledge at the time when we frame them. But some terms have been expressly framed or have been appropriated to express a small number of attributes. Such terms are monarchy, anarchy,

[2] Some logicians simply use the word 'differentia' for the purpose of expressing the distinguishing attributes, whether one or many; others, as Aristotle, would in the latter case speak of the 'differentiæ.'

[3] Whenever abstract terms are defined or divided, or occur as portions of definitions or divisions, they seem invariably to be treated as if they were common terms, and hence in the definitions of abstract terms, as in other definitions, the term defined may be regarded as a species; for the same reason, when an abstract term is divided, its relation to the dividing members may be described as that of genus to species.

triangle, square, school, monastery, &c. Our accepted definitions of these terms we may be quite sure we shall not be compelled to alter with advancing knowledge, provided at least that they continue to denote the same objects. But however full and adequate to our present state of knowledge our definitions of man, animal, vegetable, light, heat, &c., might be at the present time, they might, two hundred years hence, become most inadequate and incomplete, if not incorrect, even although the terms continued to denote the same objects or sensations. We might, for instance, discover the mode of transmission of light, or that the animal world was not separated from the vegetable by the same characteristics by which we now conceive it to be, and henceforth these discoveries would become part of the connotation of the words, or would materially alter the connotation. In the greater number of cases, therefore, we must be content to regard our definitions as provisional[4].

But not only must we in most cases be content with provisional definitions; we must often be content with definitions which, measured even by our present state of knowledge, are incomplete. It would often be most inconvenient, when it is required simply to distinguish between two terms, to give the full connotation of each.

[4] It would be a service to logicians if there were recognised names by which to distinguish between these two different classes of definitions, viz. the definitions of terms whose connotation is limited, and whose definitions are therefore final, and the definitions of terms whose connotation is unlimited, and whose definitions are therefore provisional.

In such cases, therefore, it is regarded as practically sufficient, if we refer the terms contrasted to their common genus, and specify a sufficient number of attributes to distinguish each from the other. Thus in the often-quoted example of definition, 'Man is a rational animal,' it would be absurd to suppose that 'rational' and 'animal' exhaust the connotation of 'man,' but the word 'rational' is sufficient, for popular purposes, roughly to distinguish the human race from the lower animals with which it is generally contrasted. When these *incomplete definitions*, as they are called (i.e. definitions which do not give the complete connotation of a term), are once admitted, it is obvious that our definitions of the same term may vary indefinitely according to the particular point of view from which we make them, or the science which we happen to be pursuing at the time.

An exposition of a term which gives no part of its connotation, but in lieu of it an enumeration of properties and accidents, is called a *Description*. Thus 'Man is a featherless biped[5]' would be regarded as a description, not as a definition of man. But the term 'description' is also extended to those cases in which properties and accidents are combined with a portion of the connotation, the latter being usually stated as the genus. Thus we might describe horse as an animal

[5] Here the attributive 'biped,' being used as a quasi-genus (i.e. a property or accident which, for a special purpose, is employed as a genus), is stated in the form of a substantive. This particular example of a description is, of course, absurd, and simply serves the purpose of roughly distinguishing men from birds.

which is domesticated, which has a mane and a tail, which has a high value attached to it, which is to be found in Arabia and Europe, &c., &c.; or we may take as an instance of a description Cuvier's definition of man, as a mammiferous animal having two hands. The latter instance is sufficient to shew that a description may often serve the purpose of distinguishing the term described from all other terms, and may therefore be quite adequate to fulfil the purposes of a definition for any special object.

From what has been said it will be seen that definitions and descriptions may be classified as follows:—

(1) A definition complete and final.

(2) Complete (so far as we know), but provisional.

(3) Incomplete, but sufficient to distinguish the term defined from all other terms.

(4) Incomplete, but sufficient to distinguish the term defined for the special purpose in hand.

(5) A description only, but sufficient to distinguish the term described from all other terms.

(6) A description only, but sufficient to distinguish the term described for the special purpose in hand.

Any pretended definition or description which does not fulfil one of these conditions must be rejected as having no value whatever. In testing a definition or description, we should at once reject it if it were applicable to any other term, unless we were expressly told that it was used only for the purposes of a particular science, and we found on investigation that it was distinguishable

from all other terms employed in that science. Thus, in a discussion on politics, it would be sufficient to define monarchy as the supreme government of one man, but, if speaking generally, I must add 'in a state,' for one man may be supreme governor in his family, in a school, in a religious association, &c.

The rules for a legitimate definition must depend on the kind of definition which we purpose or are compelled to employ. The following rules are common to the three first kinds of definition enumerated above:—

1. The definition must be *convertible* with the term defined, i.e. it must apply to every object denoted by the term defined, and to no other objects.
2. The definition must consist of one common term or abstract term employed as a common term (the genus), and of one or more attributives (the differentia or differentiæ).
3. Each of these terms, constituting the definition, must connote less than the term defined; otherwise, they afford no explication of it.
4. Each of these terms must have a connotation distinct from that of the others; otherwise, the definition will be redundant.

Note 1.—It may have occurred to the student to ask 'How are we to distinguish those attributes which form part of the connotation of a term, from the attributes which are inferred from them?' This is a question

which in many individual cases it would be extremely difficult to answer, and there might be great differences of opinion as to what attributes were to be regarded as differentiæ, and what as properties. It is in fact the same question as 'What are primary, and what are derived qualities?' The only general answer that can be given is that, wherever one attribute or quality can be inferred from another, it is to be regarded as a derived quality or a logical property, and that all those attributes that cannot be so derived are to be regarded provisionally as an assemblage of primary qualities or, in logical language, the connotation of the term. As our knowledge advances, it is of course possible that many of these primary qualities may be resolved into secondary, i.e. many of the supposed differentiæ may be resolved into properties.

Note 2.—The scholastic logicians distinguished between real and nominal definitions, meaning by a real definition the explication of the nature of a thing, by a nominal definition the explication of the meaning of a term. The first would give an answer to such a question as 'What *is* man?' the second to such a question as 'What do *we mean* by the term man?' As the meaning we attach to a term ought exactly to correspond with our knowledge of the primary or underived attributes of the thing, this distinction is now usually regarded as nugatory. Mr. Mill has however revived the expressions 'real' and 'nominal' definition to mark a distinction between those definitions which assume or imply the existence of the object defined

and those which carry with them no such assumption or implication. 'All definitions,' he says, 'are of names and of names only; but, in some definitions, it is clearly apparent that nothing is intended except to explain the meaning of the word; while in others, besides explaining the meaning of the word, it is intended to be implied that there exists a thing corresponding to the word.' Thus the definition of a 'horse' or a 'triangle' would be called a real definition, that of a 'centaur' a nominal definition. See Mill's *Logic*, Bk. I. ch. viii. s. 5.

Note 3.—Sometimes we may explain a term by a synonym, an etymology, or a translation, which may accidentally be better known to the hearer or reader than the term so explained. These explanations cannot be accepted as definitions in our sense of the word, inasmuch as they do not analyse the connotation of the term; but they would have been regarded as instances of nominal definitions by the older logicians.

CHAPTER VIII.

On Divisions and Classifications.

A DIVISION is an exposition of the denotation of a term. It may always be represented in the form of a proposition, of which the term divided forms the subject and the exposition the predicate.

In divisions the denotation of a term is expounded by enumerating, not the individuals (which would in most cases be impossible), but the smaller groups which the term denotes. The terms denoting these smaller groups are called the *dividing members* (*membra dividentia*) in contradistinction to the *divided term* (called by the older logicians the 'divided whole,' *totum divisum*).

It is plain that common terms, or abstract terms which are employed as common terms, are alone capable of being divided. Abstract terms and attributives have, in themselves, no denotation, while singular and collective terms are incapable of being divided into smaller groups. A collective term may however easily be transformed into a common term, and so rendered capable of being divided; thus 'the fourteenth regiment' may become 'soldiers of the fourteenth regiment,' and in this form may be divided into officers and privates, or other groups. The same remark holds good of abstract terms and attributives.

It may be laid down as a test of a logical division that the term divided must be predicable of each dividing member. Thus 'figure' is predicable alike of triangle, square, &c., 'man,' of white man and black man. In this manner it is distinguished from *partition* of a physical whole into its parts, as of 'man' into head, arms, legs, &c., or of the 'world' into Europe, Asia, Africa, and America. But this test, though sufficient to distinguish a division from a partition, is not sufficient to distinguish it from two other forms of propositions, which nevertheless we must beware of confounding with it. These are an *enumeration* of individuals, and a *distinction* of an equivocal term according to its various meanings, as e.g. of 'humanity,' according as it means human nature, the human race collectively, the virtue of being humane, or the study of polite letters. In a distinction the same definition is not predicable of each of the terms distinguished, but in a division the same definition is predicable of each dividing member.

In dividing a term into terms expressive of smaller groups (or, as they are frequently called, subject classes), I invariably try to think of some attribute which is predicable of certain members of the group, but not of others. This attribute suggests what is called the *fundamentum divisionis* or *principle of division* (i. e. some character of the group, which is a source of differences amongst its members). Thus, if asked to divide 'triangles,' it may first occur to me that some triangles have equal sides and others not; in this case the

property of a triangle that the lengths of its sides may be variously related, or, in other words, the relation of the sides, becomes the fundamentum divisionis, and I divide triangles into equilateral, isosceles, and scalene. Or it may first occur to me that some triangles are right-angled, and others not; in this case the measure of the angles becomes the fundamentum divisionis, and I divide triangles into right-angled, obtuse-angled, and acute-angled. Or, to take one more instance, if asked to divide 'governments,' I may reflect that in some governments the sovereignty is divided, in others it is placed entirely in the hands of one person or order, and I may divide 'governments' accordingly into pure and mixed, proceeding to divide pure governments, according to the number of persons in whose hands the sovereignty is placed, into monarchies, oligarchies, and democracies. Any division in which two or more fundamenta divisionis are confounded[1], or cut across one another, is called a *cross-division* and is logically inadmissible. Thus a division of triangles into isosceles, scalene, and right-angled, or of governments into monarchies, free governments, and mixed governments, or of men into Frenchmen, Asiatics, the unproductive classes, and barbarians, would be a cross-division. In the last example there is a confusion of no less than four fundamenta divisionis.

It is maintained by many logicians, and with justice,

[1] But, as in Natural Classifications (see p. 62 below), we may employ any number of fundamenta divisionis, provided that their results coincide.

that in every legitimate division (or, at least, in every division which we have sufficient reason for knowing to be legitimate), the process by which the division is arrived at, if strictly analysed, may be described as follows. Taking, for instance, 'triangles,' we reflect, say, that some are equilateral; we then divide triangles into equilateral and those which are not equilateral, and again, still thinking of equality of sides, those which are not equilateral into those which have two sides equal (isosceles), and those which have none (scalene). Similarly, taking 'men,' and thinking of race as our fundamentum divisionis, we divide mankind into Aryans and those who are not Aryans, the latter into Semites and those who are not, the latter into Turanians and those who are not, and so on, till our division is complete. This process, when formally drawn out, is called *Division by Dichotomy*, and the rule by which it proceeds is in each division to take two terms which admit no medium between them. Thus such pairs of terms as productive and unproductive, fallible and infallible, white and not white, would answer the purpose.

The divided term stands of course to the dividing members as a genus to species. We can also gather the differentia from the fundamentum divisionis. Thus, in dividing a term, we virtually define the subject terms. A pure government, for instance, may be defined as a government in which the sovereignty is undivided, and a mixed government as a government in which the sovereignty is divided. In defining also we virtually

divide. Thus the definition of man as a rational animal implies a division of animals into men and brutes, and the definition of a plane triangle as a three-sided rectilineal figure implies a division of rectilineal figures into three-sided and not three-sided (i. e. quadrilaterals and polygons).

The rules for a legitimate division may, in accordance with what has already been said, be stated as follows:—

1. Each dividing member must be a common term, or an abstract term which is used as a common term.

2. The divided term must be predicable of each dividing member.

3. The division must be *exhaustive*, i. e. the dividing members, when taken together, must be equal in extent to the term divided.

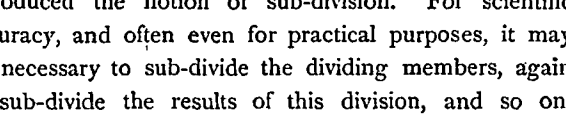
4. The dividing members must be *mutually exclusive*, i. e. no subordinate species must be included under more than one of them; otherwise we have a *cross-division*.

The surest mode of satisfying the last condition is to employ only one fundamentum divisionis, and in artificial classifications this method is adopted, but it is the distinctive characteristic of natural classifications, which alone are adequate to many of the purposes of science, to employ simultaneously a number of principles of division. (See *Inductive Logic*, ch. ii. § 2 (1), 4th ed. pp. 50–74.) In the latter case, therefore, the observation of this rule becomes a most important check on unskilful classification[2].

[2] In order to adapt the Rules of Division, as here given, to the

The student may easily frame for himself examples of illegitimate divisions. If we were absurdly to divide Europeans into Celts, Teutons, Sclaves, Frenchmen, Spaniards, the Emperor of Russia, and the President of the United States of America, we should have an example of all the faults incident to a division.

In speaking of division by dichotomy I have already introduced the notion of sub-division. For scientific accuracy, and often even for practical purposes, it may be necessary to sub-divide the dividing members, again to sub-divide the results of this division, and so on. The relation of these various divisions and sub-divisions, one to another, has given rise to several logical terms, and is of sufficient importance in itself to require a brief treatment. It is best to commence with an example, and I select a mathematical one, as being of the simplest kind:—



In a series of divisions and sub-divisions like the foregoing, the term at the head of the series (in this case Figure) is called the *Summum Genus*. The terms at the bottom of the series (equilateral triangles, circles, &c.)

Natural Classifications of Science as well as to the Artificial Classifications of ordinary life, an important change has been made in the statement of Rule 4 in the later editions of this work.

are called *Infimæ Species.* The intermediate terms are called *Subaltern Genera*, or *Subaltern Species*, viz. subaltern genera with reference to the terms immediately below them, and subaltern species with reference to the terms immediately above them. Thus triangle would be a subaltern genus with reference to equilateral triangle, a subaltern species with reference to rectilinear figure. Species which fall immediately under the same genus, as e. g. triangle, quadrilateral, and polygon, are sometimes called *Cognate Species*, but might, perhaps, with greater propriety be called *Co-ordinate Species.* A *Cognate Genus* is any one of the ascending genera under which the species falls. Thus triangle, rectilinear figure, figure, are all genera cognate to equilateral triangle. A differentia which *constitutes*, as is said, the species of which we happen to be speaking, that is, which distinguishes it from its co-ordinate species, is called a *Specific Difference*, one which constitutes any of its cognate genera is called a *Generic Difference.* Thus 'equilateral' is a specific difference, 'three-sided' a generic difference of an equilateral triangle. Or, 'three-sided' would be regarded as a specific difference, and 'rectilinear' a generic difference of a triangle. Lastly, a property derived from an attribute or attributes connoted by the species of which we happen to be speaking is called a *Specific Property*; a property which is derived from any of its cognate genera is called a *Generic Property.* It is, for instance, a property of all rectilineal figures that the sum of their angles is equal to twice as many right angles as the figure has

sides, *minus* four right angles. Thus the angles of a plane triangle are together equal to two right angles, those of a quadrilateral to four, those of a pentagon to six, and so on. It is also a property of a triangle that it may be generated by the section of a cone, but this is not a property common to other rectilinear figures. Hence the latter would be called a specific, the former a generic property.

The instance I have given is one of the simplest that could be selected. If I had taken instead of it, say, the division of animals into vertebrate and invertebrate, of vertebrate animals into fishes, amphibia, reptiles, birds, and mammals, of mammals into the various species of men, horses, oxen, &c., it would have required a long scientific discussion to distinguish the various species and genera, to state the specific and generic differences, and to give instances of specific and generic properties. And yet it is exactly in such a case as this that divisions and sub-divisions (or, in one word, Classifications) are most important. In fact, the sciences of Botany and Zoology (in the vulgar acceptation of these words) consist entirely of classifications. To give rules for so important and complicated a process as scientific classification, or even to attempt any precise definition of the word, would be to go beyond the scope of an elementary work like this[3]. It may be sufficient to suggest that where, as in the case of plants and animals, species are

[3] The reader will find a special section on Classification in the Author's 'Elements of Inductive Logic.'

separated from one another by an indefinite number of attributes, and may be separated by many attributes of which we are still ignorant, our classifications, like our definitions, should always be regarded as provisional. To this I may add two plain rules, which meet with universal acceptance: first, that our classifications should proceed as gradually as possible; and second, that we should select as our main principles of division attributes the most fruitful in their consequences, i. e. attributes from which the largest number of important properties can be derived[4]. Thus the natural system of Botany, founded, in its main division, on differences in the seed-vessels of plants, is far more instructive than the Linnæan system, founded on differences in the numbers of the pistils and stamens.

Note.—I have employed the expressions 'summum genus' and 'infima species,' as if they were entirely relative to any particular classification. But in the Isagoge of Porphyry, and by the Scholastic logicians who, for the most part, adopted his account, both summa genera and infimæ species, as well as all the subaltern genera and species, were regarded as unalterably fixed by nature. Thus the ten Categories of Aristotle (Substance, Quantity, Quality, Relation, &c.) were regarded as the summa genera, and terms like man, horse, &c. were regarded as

[4] Such attributes are called by the French physiologists *caractères dominateurs*. See Milne Edwards, *Cours Élémentaire de Zoologie*, édition septième, § 367.

expressing infimæ species. Classes like 'black men,' 'Arabian horses,' &c. would not have been admitted to be species at all. We, on the contrary, conceive that there is no limit to our power of making classes; however specialised a group may be, we can almost always think of some attribute, the addition of which will make it more special still.

PART III.—Of Inferences.

CHAPTER I.

On the Various Kinds of Inferences.

THE third and most important part of Logic treats of Inferences[1]. Wherever we assert a proposition in consequence of one or more other propositions, or, in other words, wherever we regard one or more propositions as justifying us in asserting a proposition distinct from any that has preceded, the combination of propositions may be regarded as an inference. Thus defined, inferences may be divided into inductive and deductive, and deductive inferences may be sub-divided into mediate and immediate. I shall attempt to make these distinctions clear by examples.

[1] The word 'inference' is employed in no less than three different senses. It is sometimes used to express the conclusion in conjunction with the premiss or premisses from which it is derived, as when we speak of a syllogism or an induction as an inference; sometimes it is used to express the conclusion alone; sometimes the process by which the conclusion is derived from the premisses, as when we speak of Induction or Deduction as inferences or inferential processes. Except where the meaning is obvious from the context, I shall endeavour to confine the word to the first-named signification. The terms *Induction* and *Deduction* will be appropriated to express *processes* which result, the former in *inductions* or *inductive inferences*, the latter in *deductions* or *deductive inferences*, these last being sub-divided into *syllogisms* and *immediate inferences*.

I mix tartaric acid and carbonate of soda in certain proportions in water, and I observe that the mixture is followed by an effervescence; from this I *infer* that, whenever tartaric acid and carbonate of soda are mixed in water in these proportions, effervescence will follow. I put a poker into the fire, and I observe that after a time it becomes red-hot; from this I *infer* that incandescence can always be produced in iron by a certain degree of heat. I observe five points in the orbit of a planet, and, from my knowledge of mathematics, perceive that they are situated in an ellipse: from this I *infer* that the entire orbit of the planet is elliptical, and that, in all future revolutions of the planet, a similar orbit will be described. Now what in these cases do I mean by the word 'infer?' That the mixture is *followed* by effervescence is a matter of observation; but it is only an inferential process which justifies me in asserting that, inasmuch as it could have been produced by nothing else, the effervescence was *produced* by the mixture, and that, whenever in future I see a similar mixture, I may expect to see it followed by similar results. Two assumptions, it will be seen, underlie this inference: 1st, that every event has a cause, which leads me to assume that the effervescence must have been produced by some cause or other; 2nd, the belief in the uniformity of nature, which leads me to expect that, whenever similar circumstances are repeated, they will be followed by similar results. The reasoning therefore in these cases may be represented as follows:—

The mixture of the tartaric acid with the carbonate

of soda is *followed* by effervescence. (Original Proposition.)

∴ (Owing to the special circumstances of the case and in accordance with the principle that every event must have a cause), the effervescence was *produced* by the mixture.

∴ (In accordance with the principle of the uniformity of nature), a similar mixture will always be followed by an effervescence.

We may represent the reasoning in the third example in the same manner:—

We may assert (by virtue of our knowledge of mathematics) that five points which we have observed in the orbit of the planet Mars are situated in the arc of an ellipse. (Original Proposition.)

∴ As there are, comparatively speaking, no causes determining the position of the planet at any given moment, except the attraction of the sun and the continued effects of the initial velocity, we may infer that the fact of the five points observed being situated in the arc of an ellipse is due to the combination of these two causes.

∴ (In accordance with the principle of the uniformity of nature), it may be inferred that all other points in the orbit of the planet are situated in an ellipse, and that, in all future revolutions, a similar orbit will be described; i. e. the orbit of the planet Mars may be regarded as elliptical.

Now inferences of this kind are called *Inductive.* The instances I have selected are remarkably simple, but they are sufficient to shew that an induction may be defined as an inference in which we argue from particulars to adjacent particulars, or (if we speak of the adjacent particulars collectively) from particulars to universals, in accordance with the laws of universal causation and of the uniformity of nature. As to the circumstances which justify us in asserting that one phenomenon or set of phenomena is the cause or the effect of, or is invariably conjoined with, another (for this is the problem of Induction), the student is referred to works specially treating of Inductive Inference. It is sufficient here to distinguish inductive from the deductive inferences which it is our more special business to explain[2].

[2] An *Analogy* is a form of imperfect induction, and, though justifying a conclusion more or less probable, never leads to certainty. If two objects resemble each other in several important respects, and we argue that any particular attribute which we know to be predicable of the one, and do not know to be either predicable or not predicable of the other, is, on account of the general resemblance of the two objects, also predicable of the other, the argument is called an argument from analogy; and, in the same way, if two objects are dissimilar, we may argue that an attribute which is predicable of the one, is, on account of their dissimilarity, not predicable of the other Thus, from the similarity between the earth and the moon, we might argue that the latter is inhabited, or, from their dissimilarity, that it is not inhabited. The value of the inference always depends on the ratio of the ascertained resemblances to the ascertained differences (it being understood that the resemblances which we take into account are none of them derived, as properties, from each other, and so with the differences), providing that our knowledge of the objects is sufficiently large to justify us in drawing any inference at all. For a more

Beginning where induction ended, we may state such a proposition as this: 'All iron when heated to a certain degree becomes red-hot.' This, if combined with another proposition 'This is a piece of iron,' leads to the conclusion 'This piece of iron, if heated to a certain degree, will become red-hot.' Now it is plain that the con-

detailed analysis of this mode of reasoning, and an estimate of the value to be attached to its conclusions, the student is referred to the author's *Elements of Inductive Logic*, ch. iv., and to Mr. Mill's *Logic*, Bk. III. ch. xx., one of the most instructive and important chapters in his work.

It should be noticed that an analogy, as here described, is not identical with the Analogy of Aristotle, the Aristotelian Analogy being an equality of relations (*ἰσότης λόγων*). Thus the expression,

The intellect: the soul = the sight: the body,

is an *ἀναλογία*. From this analogy it is argued that anything which may be predicated of the one pair of terms may be predicated also of the other. Or, to take a non-Aristotelian instance, which will be more intelligible to beginners:

A colony: the mother-country = a child: a parent.

From this analogy it is argued that the reciprocal rights and duties of a colony and the mother-state are the same as those of a child and a parent. In this form of argument, if the relations between the two sets of terms were precisely the same in all respects, the conclusion would be invariably valid; as it is usually found in practice, however, the relations are the same in some respects, but not in others, and, consequently, the conclusion is valid, when based on those points in which the relations are the same, and invalid, when based on those points in which the relations are not the same. Thus, it might be maintained that, in many respects, the relation of the child to the parent is not the same as that of a colony to the mother-country, and, hence, that many of the rights and duties which exist in the one case do not exist in the other. A *Metaphor* is an analogy of this kind compressed into a single word or phrase.

The fallacy of False Analogy will be noticed below.

clusion we have just drawn was arrived at in a different manner from those noticed above. Instead of being the conclusion of a process by which we argue from particulars to adjacent particulars or from particulars to universals (i. e. from cases which are within the range of our observation to others which are without), it is simply a combination of two propositions into one, being an obvious inference from what has been previously stated in the premisses. Induction has been not inaptly compared to the establishing of a formula, Deduction (for that is the appropriate name of the process which I am now discussing) to the reading it off. Induction leads to truths entirely new, Deduction combines, methodizes, and developes those which we have already gained.

A Syllogism may be called a Mediate Inference, because the two terms of the conclusion are compared in the premisses by means of a third. It is thus opposed to an Immediate Inference, which consists of two propositions only, and in which the inferred proposition is derived from a single proposition without the aid of any other term or proposition, expressed or implied. Both mediate and immediate inferences may be styled deductive as opposed to inductive.

This division may easily be shewn to be exhaustive. In any inference, we argue either to something already implied in the premisses[3] or not; if the latter, the

[3] If we state explicitly all the assumptions made in the inductive process, the conclusion is contained in the premisses, and the form of the reasoning becomes deductive; but it is seldom that we do state

inference is inductive, if the former, deductive. If the deductive inference contain only a single premiss, it is immediate; if it contain two premisses and the conclusion be drawn from these jointly, it is mediate and is called a syllogism. All deductive inferences which apparently contain more premisses than two admit of being analysed into a series of syllogisms.

Note 1.—I am here departing from the ordinary scheme of division adopted by logicians. Inferences are generally divided into mediate and immediate, and mediate inferences are subdivided into inductive and deductive. As however I regard inductions as more strongly contrasted with both syllogisms and immediate inferences than either of these classes is with the other, it seems preferable to make inductions one of the main members, rather than

our assumptions thus explicitly. The most essential distinction, however, between inductive and deductive reasoning consists not in the form of the inferences, but in the nature of the assumptions on which they rest. Deductive reasoning rests on certain assumptions with regard to language and co-existence (namely, the so-called Law of Identity, or some modification of that law, the Law of Contradiction, the Law of Excluded Middle, and the Canons of Syllogism), while inductive reasoning assumes over and besides these laws the truth of the Laws of Universal Causation of the Uniformity of Nature and, in certain cases, of the Conservation of Energy; or, if it be of the unscientific description which is known as Inductio per Enumerationem Simplicem, it merely assumes, instead of them, the vague and wide principle that the unknown resembles, or will resemble, the known. It hardly needs to be added that all reasoning alike assumes the trustworthiness of present consciousness and of memory.

one of the subordinate members of the division. Nor is there any reason why an immediate inference should not be regarded as deductive.

It should also be noticed that Sir W. Hamilton would deny the title of inferences to inductions (as they have been here explained), whereas Mr. Mill would deny that either a syllogism or an immediate inference can properly be called an inference. Mr. Mill maintains that all Inference is 'from the known to the unknown'; Sir W. Hamilton defines Inference as the 'carrying out into the last proposition what was virtually contained in the antecedent judgments.'

Note 2.—The Aristotelian induction, in which the conclusion affirms or denies of a group what was in the premisses affirmed or denied of each member of the group severally, is, according to the above method of treatment, obviously regarded as a deductive inference. If I predicate some quality of each member of a group, and thence infer that all members of the group possess this quality, the conclusion is plainly contained in the premisses, and the inference is a syllogism. It may be represented in the form[4]—

[4] By Aristotle himself the inductive inference is analysed thus :—

x, y, z are B,
x, y, z are (i.e. constitute) A ;
∴ A is B.

The minor premiss, when stated in so peculiar a form, of course admits of simple conversion, and thus assumes the form given in the text.

x, y, z are B,
The individuals (or subordinate species) constituting the group A are x, y, z;
∴ The individuals (or subordinate species) constituting the group A are B.

Such an inference is altogether different from what we now understand by an induction. On this subject the student may with advantage read Mr. Mill's chapter on 'Inductions improperly so called.' See Mill's *Logic*, Bk. III. ch. ii. An account of the Aristotelian induction will be found in Appendix G to Dr. Mansel's edition of *Aldrich;* in Sir W. Hamilton's *Essay on Logic*, and in his *Lectures on Logic*, Lect. xvii. and Appendix vii. These authors, as already noticed in the case of Sir W. Hamilton, regard inductions, in the modern sense of the word, as extra-logical. The advanced student may also consult with advantage Mr. De Morgan's chapter on 'Induction,' *Formal Logic*, ch. xi.

CHAPTER II.

On Immediate Inferences.

§ 1. AN Immediate Inference may be formally defined as a combination of two propositions of which one is inferred from the other, the proposition inferred being virtually included in the proposition from which it is inferred. Of Immediate Inferences the most important forms are Oppositions, Conversions, Permutations[1].

§ 2. *On Oppositions.*

Two propositions are said to be *opposed* when they have the same subject and predicate, but differ in quantity or quality or both. An Opposition may be defined as an immediate inference in which from the truth or falsity of one proposition we infer either the truth or falsity of another, this proposition having the same subject and predicate as the former, but differing in quantity or quality or both. Thus from the proposition 'That all X is Y is true' we may infer the proposition 'That no X is Y is false,' or 'That some X is Y is true,' or 'That some X is not Y is false.'

[1] It is the more common practice to speak of Opposition, Conversion, and Permutation, but I have adopted the plural number in order to draw attention to the fact that Logic is concerned with the results rather than with the processes by which they are arrived at.

The opposition between

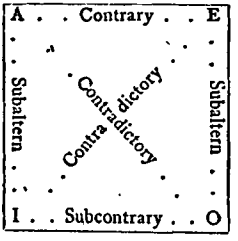
A and E is called a Contrary Opposition.

I and O, a Subcontrary Opposition.

A and I, or E and O, a Subaltern Opposition.

A and O, or E and I, a Contradictory Opposition.

These forms of Opposition are exhibited in the annexed scheme :—



If A be true; E is false, I true, O false.

If A be false; E is unknown, I unknown, O true.

If E be true; A is false, I false, O true.

If E be false; A is unknown, I true, O unknown.

If I be true; A is unknown, E false, O unknown.

If I be false; A is false, E true, O true.

If O be true; A is false, E unknown, I unknown.

If O be false; A is true, E false, I true.

It will be observed that it is only in a Contradictory Opposition (where the opposed terms differ both in quantity and quality) that from the truth or falsity of one proposition we can invariably infer the truth or falsity of another, the conclusion which we draw in this case being from the truth or falsity of the one propo-

sition to the falsity or truth respectively of the other. Hence logicians have called contradictory the most perfect form of opposition. It is a rule of practical Logic that a contradictory should always in disputations be used in preference to a contrary opposition; for it serves equally well the purpose of contradicting an opponent, and the particular proposition which it asserts affords less ground for attack than an universal. Thus, if my opponent asserts A (as e.g. All philosophers are unimaginative), I may meet his assertion by the contradictory O (Some philosophers, as e.g. Plato, Goethe, &c., are not unimaginative), and from this position I cannot well be dislodged. But suppose I assert in opposition to him an E proposition (No philosophers are unimaginative), he will probably be able to adduce instances of some philosophers who, according to the ordinary meaning of the word 'imaginative,' would be called unimaginative, and so, by meeting my E with an I proposition, gain an apparent victory. As a fact, we should each have made assertions too wide, but he would have succeeded in dislodging me from my position, whereas (owing to my neglect of the laws of contradiction) I should not have succeeded in dislodging him from his.

Note.—It is plain that, according to the ordinary meaning of the word 'opposition,' it is somewhat of an abuse of language to speak of A and I, or E and O propositions as being opposed. It would be better

if this form of inference were called Subalternation or Subordination.

Nor, strictly speaking, can the relation between I and O be called one of opposition, for they may both be true together. Accordingly, Aristotle says that in reality (κατ' ἀλήθειαν) there are three forms of opposition (those between A and E, A and O, E and I), though in language (κατὰ τὴν λέξιν) there are four (adding that between I and O). What is called Subaltern Opposition he does not recognise.

§ 3. *On Conversions.*

A proposition is said to be *converted* when its terms are transposed, so that the subject becomes the predicate, and the predicate the subject. A Conversion may be defined as an immediate inference in which from one proposition we infer another having the same terms as the original proposition, but their order reversed. This inference in some cases necessitates a change of quantity in passing from one proposition to the other, and then it is called a *Conversion per accidens*; when it necessitates no such change, it is called a *Simple Conversion*[2].

I and E may both be converted simply. Thus, from 'Some X is Y,' or 'Some poets are philosophers,' I may infer 'Some Y is X,' or 'Some philosophers are poets.' From 'No X is Y,' or 'No savages are trustworthy,' I may infer 'No Y is X,' or 'No trustworthy persons are savages.'

[2] It is proposed by Sir W. Hamilton to call the original proposition the 'Convertend,' the inferred proposition the 'Converse.'

A can only be converted *per accidens*. For though it may sometimes happen that the subject and predicate of an A proposition are co-extensive, and therefore convertible, this circumstance is not implied in the form of the proposition, and it is with what is implied in the form of the proposition that we are alone concerned. Thus, if I assert the proposition 'All plane triangles are three-sided rectilineal figures,' it happens in this particular case that I am justified, without any change of quantity, in stating the converse, 'All three-sided rectilineal figures are plane triangles.' But if I state that 'All plane triangles are rectilineal figures,' I am only justified in inferring that 'Some rectilineal figures are plane triangles.' As, therefore, the general form of an A proposition does not imply the simple convertibility of the subject and predicate, I am only justified in inferring from 'All X is Y,' that 'Some Y is X.'

In those cases, however, in which the form of the proposition implies that the subject and predicate are co-extensive, the proposition, though an A proposition, may be converted simply. Thus, from the propositions 'The second legion is the only legion quartered in Britain,' 'Virtue is the condition of Happiness,' 'All plane triangles may be defined as three-sided rectilineal figures,' it may be inferred by simple conversion that 'The only legion quartered in Britain is the second legion,' 'The condition of Happiness is Virtue,' 'All three-sided rectilineal figures may be called plane triangles.'

An O proposition cannot be converted at all. From

'Some X is not Y,' it does not follow that 'Some Y is not X,' for Y may stand to X in the relation of a species to a genus. Thus from the proposition 'Some Europeans are not Frenchmen,' I cannot infer that 'Some Frenchmen are not Europeans.'

§ 4. *On Permutations*[3]

A Permutation may be defined as an immediate Inference in which from one proposition we infer another differing in quality, and having, therefore, instead of the original predicate its contradictory. Thus:—

From All X is Y, we may infer that No X is not-Y.
From No X is Y, ... All X is not-Y.
From Some X is Y, ... Some X is not not-Y.
From Some X is not Y, . . . Some X is not-Y.

The legitimacy of these inferences is apparent from the fact that contradictory terms (A and not-A) admit of no medium, so that, if I predicate the one affirmatively, I may always predicate the other negatively, and vice versâ.

The O proposition, when permuted from 'Some X is not Y,' into 'Some X is not-Y,' may of course be converted into 'Some not-Y is X.' This combination of permutation and conversion is improperly described by Whately and many previous logicians as a single inference, and styled 'Conversion by Contra-Position or Negation.'

[3] The term Permutation is borrowed from Mr. Karslake's *Aids to Logic*. The same inference is sometimes called Infinitation, from the Nomen Infinitum, or, more properly, Nomen Indefinitum (not-Y, as the contradictory of Y), which is employed as the predicate.

It may assist the student if I add some further instances of permutations :—

All men are fallible, ∴ No men are infallible.

No men are infallible, ∴ All men are fallible.

Some poets are reflective, ∴ Some poets are not unreflective.

Some poets are not unreflective, ∴ Some poets are reflective.

All poets are men of genius, ∴ (by permutation) No poets are not-men-of-genius; ∴ (by conversion) No not-men-of-genius (=None but men of genius) are poets.

Note.—I have here employed an expression 'Contradictory Terms,' which in most works on Logic is explained in the first part, as included under the doctrine of Opposition of Terms. It seemed, however, desirable to introduce there only those distinctions of terms which were likely to be frequently required in the sequel of the work. I may here state that 'Contradictory Terms,' such as white and not-white, lawful and un-lawful, are terms which admit of no medium, i.e. terms which are not both predicable of the same thing, while one or other of them must be predicable of it. 'Contrary Terms,' like good and bad, black and white, are terms which are the most opposed under the same genus; they cannot both be predicated of the same thing, but it is not necessary that one or other of them should be predicable of it.

CHAPTER III.

On Mediate Inference or Syllogism.

§ 1. *The Structure of the Syllogism.*

A SYLLOGISM may be defined as a combination of two propositions, necessitating a third in virtue of their mutual connexion; or as an inference in which one proposition is inferred from two others conjointly, the inferred proposition being virtually contained in the propositions from which it is inferred. This is obviously a definition of a legitimate syllogism. There may (as will appear below) be apparent syllogisms, which do not fulfil the conditions of this definition. We may take as instances of syllogisms:—

(1) All B is A,
All C is B;
∴ All C is A.

(2) All sovereign powers are invested with supreme authority over their subjects,
All republics are sovereign powers;
∴ All republics are invested with supreme authority over their subjects.

(3) No rectilinear figure is bounded by one line,
A circle is bounded by one line;
∴ A circle is not a rectilinear figure.

The proposition inferred is called *the Conclusion*, the propositions from which it is inferred *the Premisses*, either of them singly being called *a Premiss*.

As the conclusion is virtually contained in the premisses conjointly, it is plain that the two terms of the conclusion must occur in the premisses, one in either. If both terms occurred in the same premiss, the other premiss would be entirely alien to the conclusion. The remaining term of each premiss must be the same; else there would be nothing in common between the two premisses, and the conclusion could not be said to be inferred from the two conjointly. This third term, with which the two terms of the conclusion may be regarded as compared, is called the *middle term*. The predicate of the conclusion is called the *major term*, and the subject the *minor term*; the premiss, in which the major and middle terms are compared, is called the *major Premiss*, and should always be stated first; that in which the minor and middle terms are compared is called the *minor premiss*. Thus in a syllogism, formally stated, there are always three propositions including three terms, the premisses occurring first and the conclusion last. But practically, in reasoning, we frequently state the conclusion first, introducing one or both premisses with such a word as 'for' or 'because,' as stating our reason for the assertion. Thus I may say 'I will not go out to-day, for it is raining,' or, 'I will not go out to-day, for it is raining, and the rain may give me a cold.' When stated in this form, the conclusion is called by the older logicians the *Problema*

or *Quæstio*, being regarded as a question to which the reason or reasons assigned furnish the answer. It will also have occurred to the student that, as a fact, we usually state only one premiss, leaving the other (which may be either the major or minor) to be understood. Thus, instead of stating Syllogism (2) formally, as above, I should in an actual discussion say, 'A republic is invested with supreme authority over its subjects, for every sovereign power is invested with such authority,' or, 'A republic is, &c., for it is a sovereign power,' or briefly, 'A republic (being a sovereign power) is invested,' &c.; or, the premiss coming first, 'Inasmuch as every sovereign power is invested, &c., I maintain a republic to be invested with that authority,' or, 'Inasmuch as a republic is a sovereign power, it is invested with,' &c.[1] Instead of suppressing one of the premisses, I may, for brevity's sake, suppress the conclusion. Thus I may say 'Every sovereign power is invested with supreme authority over its subjects, and a republic is a sovereign power,' leaving it to the hearer or reader to draw the conclusion for himself. The syllogism does not pretend to be the form, or even a form, in which our reasonings are usually stated, but simply one of the ultimate analyses of them.

As every term in the syllogism occurs twice, it should be noticed that, on both occasions, it should be used in the same sense, or, to adopt technical language, every

[1] A syllogism with a suppressed premiss is by Aldrich wrongly identified with the Enthymeme of Aristotle. Such a syllogism was called by the Stoics a *συλλογισμὸς μονολήμματος*.

term in the syllogism should be used *univocally*. If we use a term *equivocally*, i.e. in two entirely different senses, or even *analogously*, i.e. in two different senses having some relation to each other, it is plain that, logically speaking, we are using two different terms, and consequently the syllogism will include four terms instead of three. This caution includes the rule usually given by logicians against an *ambiguous middle*. The neglect of it, palpable as it might be supposed to be, is often, especially in a long course of reasoning, very difficult of detection, and is a fertile source of fallacy. I may adduce as very simple instances:—

Humanity is a moral virtue,
The study of polite letters is humanity;
∴ The study of polite letters is a moral virtue.

The church is the aggregate of all Christian people,
This particular congregation (or particular building) is the church (meaning at some particular place);
∴ This particular congregation (or particular building) is the aggregate of all Christian people.

In the former case, the term 'humanity' has come to be used in such widely different senses, that it may be regarded as used equivocally; in the latter case, the senses of the word 'church' are perhaps sufficiently nearly allied to be regarded as analogous. All cases of what are termed 'Verbal Fallacies' may be referred to this head.

Note.—The words 'major,' 'minor,' and 'middle,' as applied to the terms in a syllogism, have been inherited

by all subsequent logicians from the nomenclature of Aristotle. He regarded what we shall presently call the First Figure (B is or is not A, C is B, ∴ C is or is not A) as the perfect type of syllogism, and, amongst other modes, stated it in the form C is in B (*τὸ Γ ἐστὶν ἐν ὅλῳ τῷ* B), B is or is not in A, ∴ C is or is not in A. Thus stated, C appears to be the smallest, B the intermediate, and A the largest term in extent. See *Prior Analytics*, Bk. I. ch. iv. In negative propositions, however, we have no means of determining the relative extent of the subject and predicate, and consequently Aristotle's nomenclature does not properly apply to negative syllogisms. To affirmative syllogisms in the first figure, whether universal or particular, it applies only in a modified shape, for the propositions All X is Y, Some X is Y, though they imply that Y cannot be less in logical extent than All X in the one case or than Some X in the other, do not exclude the possibility of the subject and predicate being co-extensive. Hence, however convenient it may be, Aristotle's nomenclature applies only, and that not with strict accuracy, to two forms of syllogism (Barbara and Darii) in the first figure.

§ 2. *On Moods and Figures.*

I now proceed to consider the possible, not the legitimate, forms of syllogism. Here there are two circumstances to be taken into consideration: 1st, that syllogisms may vary according to the quantity and quality

of the propositions (A, E, I, O) of which they are composed; 2nd, that they may vary according to the position of the terms in the premisses. The first consideration gives us the number of possible *moods*, the second the number of possible *figures*. It is by combining these two sources of variation that we shall obtain the number of possible syllogisms.

There are, if we take into consideration the conclusion, sixty-four possible arrangements of the propositions A, E, I, O, i. e. in technical language, sixty-four possible *moods*, viz. AAA, AAE, AAI, AAO, &c. But if we consider the premisses only, the number of possible moods is limited to sixteen, viz. AA, AE, AI, AO, EA, EE, EI, EO, IA, IE, II, IO, OA, OE, OI, OO. In determining what possible moods are legitimate, we may either ask 'Is this conclusion justified by these premisses?' or 'To what conclusion do these premisses lead?' If we ask the former question, we must examine the sixty-four possible moods in which the conclusion appears as well as the premisses; if the latter, an examination of the sixteen possible arrangements of premisses is sufficient.

With respect to the possible arrangements of the terms in the premisses (i. e. the *figures*, as they are technically called) there are also two methods of proceeding. Taking no account of the conclusion (and therefore not knowing which is the major term and which the minor), and asking simply 'In how many ways can the middle term be combined with the other terms in the premisses?' there are three possible figures; viz. 1st, that in which the

middle term is subject in one premiss and predicate in the other; 2nd, that in which it is predicate in both premisses; 3rd, that in which it is subject in both. But if we take account of the conclusion, we are able to distinguish the major and minor terms, and consequently the major and minor premisses. In this case, there are four possible figures, viz. 1st, that in which the middle term is subject in the major premiss and predicate in the minor; 2nd, that in which it is predicate in both premisses; 3rd, that in which it is subject in both; 4th, that in which it is predicate in the major premiss and subject in the minor. These four figures may be exhibited thus:—

Fig. 1.	Fig. 2.	Fig. 3.	Fig. 4.
BA	AB	BA	AB
CB	CB	BC	BC
∴ CA	∴ CA	∴ CA	∴ CA.

If we take no account of the conclusion, either extreme in the premisses may become the major term, and the three figures may be represented thus:—

Fig. 1.	Fig. 2.	Fig. 3.
BA	AB	BA
CB	CB	BC
∴ CA or AC	∴ CA or AC	∴ CA or AC.

§ 3. *Determination of the Legitimate Moods of Syllogism.*

Note.—Few difficulties in elementary Logic are more likely to embarrass the beginner than the variety of methods of constituting the Legitimate Moods of Syllogism. Sir W. Hamilton, as a consequence of quantifying

the predicate, is able to represent all syllogisms as equations, and thus to exhibit every affirmative syllogism as a direct application of what is called the Law of Identity (Every A is A), and every Negative Syllogism as a direct application of the Law of Contradiction (No A is not-A)[2]. Besides Sir W. Hamilton, other logicians who do not, like him, quantify the predicate, have also attempted to enunciate general principles equally applicable to all syllogisms. See e.g. *Port Royal Logic*, Part III. ch. x. ‹ Others (as Abp. Thomson, *Laws of Thought*, § 96, and Lambert, as quoted by Dr. Mansel in his *Notes on Aldrich*, ch. iii. § 6) enunciate a distinct principle for each figure. Others (and pre-eminently Aristotle) enunciate a canon for the first figure[3], and test

[2] These two Laws (or Principles) together with the Law or Principle of Excluded Middle (A either is or is not B) and the Law or Principle of Sufficient Reason (Infer nothing without a ground or reason), are often called the Fundamental Laws of Thought. See Thomson's *Laws of Thought*, § 114, and Hamilton's *Lectures on Logic*, Lects. vi. and vii.

[3] *Λέγομεν δὲ τὸ κατὰ παντὸς κατηγορεῖσθαι, ὅταν μηδὲν ᾖ λαβεῖν τῶν τοῦ ὑποκειμένου, καθ' οὗ θάτερον οὐ λεχθήσεται· καὶ τὸ κατὰ μηδενὸς ὡσαύτως*, An. Pr. I. 1; *ὅσα κατὰ τοῦ κατηγορουμένου λέγεται πάντα καὶ κατὰ τοῦ ὑποκειμένου ῥηθήσεται*, Cat. 3. This principle or canon in its Latin form was called by the Schoolmen the *Dictum de Omni et Nullo*. It is thus stated by Sanderson: 'Dictum de Omni est hujusmodi: Quidquid affirmatur Universaliter de aliquo Subjecto, affirmari necesse est de iis quæ sub eo continentur. Dictum de Nullo hujusmodi: Quidquid negatur Universaliter de aliquo Subjecto, negari necesse est de iis quæ sub eo continentur.' By Aldrich it is stated in a more compressed form, as follows: 'Quod prædicatur Universaliter de allo (i.e. de termino distributo), sive affirmative, sive negative, prædicatur similiter de omnibus, sub eo contentis.'

the validity of syllogisms in all other figures by reducing them to the first. Lastly, a favourite method amongst logicians is to enumerate the faults which are incident to a syllogism, and then reject those moods in which they are found. This method is often combined with one or more of the others. Aldrich, for instance, enunciates general canons of syllogism, then uses the method I have last explained, and finally reduces syllogisms in the other figures to their corresponding forms in the first.

In accordance with the ordinary practice of elementary treatises, and as being perhaps at first more intelligible to the learner, I shall take into consideration the conclusion, and consequently regard the number of figures as four, and that of possible moods as sixty-four, reserving for a note the shorter and more scientific procedure. The problem therefore now before us is to determine which of the sixty-four moods are admissible in each of the four figures.

In the first figure our task is easy. There we are able to establish a canon which will determine directly the legitimate moods.

With a little attention the student will be able to perceive the truth of the following propositions:—

(α) If one term be affirmed of another (provided the latter be taken in its entire extent), and this term of a third, the first may be affirmed of the third.

(β) If one term be denied of another (provided the

latter be taken in its entire extent), and this term affirmed of a third, the first may be denied of the third.

(γ) If one term be affirmed of another (even though the latter be taken in its entire extent), and this term denied of a third, we are not justified in drawing any conclusion as to the relation of the first to the third. For, if one term be denied of another, it does not follow that whatever may be predicated of this first term may also be denied of the other: thus I may deny 'red' of 'blue,' but it does not follow that 'colour,' which I predicate of 'red,' may also be denied of 'blue.'

(δ) If one term be denied of another (even though the latter be taken in its entire extent), and this term denied of a third, no conclusion can be drawn as to the relation of the first to the third. For, because I deny one term of another, it does not follow that whatever I can deny of the first can also be denied of the other, nor does it follow that it can be affirmed of it: thus, because I can deny 'white' of crows, and 'black' of 'white' (or rather of the corresponding common term 'white things'), it does not follow that I can deny 'black' of crows; nor, because I can deny 'white' of crows as well as 'yellow' of 'white' (or 'white things'), does it follow that I can affirm 'yellow' of crows.

Putting together these results, we obtain the following

canon of reasoning in the first figure: If one term can be affirmed or denied of another (provided that the latter be taken in its entire extent), and this term affirmed of a third, the first can be affirmed or denied (respectively) of the third; and, if these conditions are not fulfilled, no conclusion can be drawn[4]. The only moods which fulfil the conditions of the canon are AAA, EAE, AII, and EIO. The conclusions of two other moods, namely, AAI and EAO, might be inferred, by Subaltern Opposition, from those of AAA and EAE, and hence these are called *Subaltern Moods.*

We have obtained, it will be observed, forms of syllogism capable of proving any one of the four propositions, A, E, I, or O, and into one or other of the types accepted as legitimate moods of the first figure all our mediate reasonings may be thrown. Here, then, our enquiry might terminate, if it were simply our object to obtain a sufficient number of legitimate types of reasoning, but the problem before us is to state exhaustively all possible forms which can be accepted as legitimate.

There being no canon which distinguishes with equal precision the legitimate and illegitimate moods of the other figures, we must, in discussing them, have recourse to some other method. I shall first enumerate and explain certain syllogistic rules (derived from the definition

[4] It will be plain from the statement of the Canon that the major premiss of a syllogism in the first figure must be universal. At the suggestion of Mr. St. G. Stock of Pembroke College, this fact has been made plainer in the statement of the Canon itself than was the case in the earlier editions.

of a syllogism) which will exclude illegitimate moods, and then, before accepting the remainder, I shall test them by reducing them to the first figure.

Syllogistic Rules.

I. *The middle term must be distributed at least once.* For, if in both premisses it were used in only a partial signification, it might denote entirely different objects in the one premiss from those which it denoted in the other, and so there might be no connexion between the two premisses. Thus, in the premisses 'All men are animals,' 'All horses are animals,' the part of the group 'animals' which is coincident with 'men' may be, and here is, entirely distinct from that portion of the group which is coincident with 'horses,' and consequently we can draw no conclusion as to the relation between men and horses. This fallacy is called *Undistributed Middle.*

II. *If a term be distributed in the conclusion, it must have been previously distributed in the premisses.* The reason is obvious. If we use a term in a partial signification in the premisses, we cannot legitimately use it in its entire signification in the conclusion. To do so would be to argue from part to whole, or, in other words, to employ a term in a wider signification in the conclusion than that in which it is employed in the premisses.

This fallacy is called *illicit process* of the major or minor, according as the term illegitimately distributed in the conclusion is the major or minor term. In the syllogism.

Some A is not B,
All B is C,
∴ Some C is not A,

we have illicit process of the major; in the syllogism,

All B is A,
Some C is B,
∴ All C is A,

illicit process of the minor.

III. *Two negative premisses prove nothing.* For they simply assert that there is no connexion between the middle term and the extremes; consequently we can draw no conclusion with respect to the relation of the extremes.

IV. *If either of the premisses be negative, the conclusion must be negative.* For the other premiss is affirmative, and, if in one premiss we affirm a connexion between the middle term and one of the extremes, and in the other premiss deny any connexion between the middle term and the other extreme, there can be no connexion between the two extremes.

V. *If the conclusion be negative, one of the premisses must be negative.* For we cannot deny that there is any connexion between the extremes, except we have previously denied that there is any connexion between one of the extremes and the middle term.

VI. *Two particular premisses prove nothing.* For they cannot be both negative (O, O). Nor can they be both affirmative (I, I), for then the middle term would be undis-

tributed. The only remaining case is that of one affirmative and one negative premiss (I, O). But this combination of premisses would leave no term to be distributed in the conclusion. Hence the conclusion would be an I proposition, an affirmative conclusion inferred from a negative premiss, which (according to Rule IV) is illegitimate.

VII. *If one premiss be particular, the conclusion must be particular.*

The premisses must be either both affirmative, or one affirmative and one negative (see Rule III).

Now, if they are both affirmative, they will (by Rule VI) be A and I. This combination of premisses, as it contains only one distributed term, and the middle term must be distributed at least once (Rule I), leaves no term to be distributed in the conclusion: consequently the conclusion must be I.

If the premisses are one affirmative and one negative, they must (by Rule VI) be either O and A or I and E. In either case the premisses distribute only two terms, and, as one of them must be the middle term (by Rule I), there remains only one term to be distributed in the conclusion. But the conclusion must be negative (by Rule IV). Therefore, it must be O [5].

The converse of this Rule, viz. that a particular conclusion necessitates a particular premiss, is not true. The only cases however in which we find a particular

[5] This proof is substituted for the somewhat more elaborate proof given in the earlier editions.

conclusion without a particular premiss are those in which the premisses assume more than is required in order to prove the conclusion. This fact will be apparent to the student from an examination of the individual cases, and it might be laid down as a rule that, wherever there is a particular conclusion without a particular premiss, something superfluous is invariably assumed in the premisses [6].

Of the above Rules, it is plain that Rules III, IV, V, VI, VII are applicable to the moods before they are referred to the several figures, Rules I and II are applicable only when the moods are referred to some particular figure.

By the application of the first set of Rules, the sixty-four possible moods are reduced to twelve, viz.

AAA, AAI, AEE, AEO, AII, AOO,
EAE, EAO, EIO, IAI, IEO [7], OAO.

Thus EEE is rejected because it has two negative premisses, EAA because it has a negative premiss without a negative conclusion, AAE because it has a negative

[6] The syllogistic rules are comprised in the mnemonic lines:—
Distribuas medium; nec quartus terminus adsit.
Utraque nec præmissa negans, nec particularis.
Sectetur partem conclusio deteriorem.
Et non distribuat, nisi cum præmissa, negetve.

[7] It has been suggested to me by Professor Park that IEO might be rejected by the application of Rule II, even before it is referred to any particular figure. For the conclusion, O, distributes its predicate, that is the major term of the syllogism. But the major premiss, being I, evidently does not distribute this term. Consequently, there is invariably Illicit Process of the Major.

conclusion without a negative premiss, III because it has two particular premisses, IAA because it has a particular premiss without a particular conclusion.

By the application of Rules I and II to these twelve moods, when referred to the several figures, there remain:—

in fig. 2, EAE, AEE, EIO, AOO, EAO, AEO;
in fig. 3, AAI, EAO, IAI, OAO, AII, EIO;
in fig. 4, AAI, AEE, IAI, EAO, EIO, AEO.

I append a few examples of the method of testing the moods, when referred to the figures.

Take AEE in figure 2.

A All A is B,
E No C is B;
E ∴ No C is A. No fault.

Take IEO in figure 3.

I Some B is A,
E No B is C;
O ∴ Some C is not A.
Illicit process of major.

Take AII in figure 4.

A All A is B,
I Some B is C;
I ∴ Some C is A. Undistributed middle.

It will be seen that of the sixty-four moods, when referred to the four figures, there are only six in each which have not been rejected. It now remains further to test

these moods in the second, third, and fourth figures by reducing them to moods in the first.

Reduction.

As we have adopted no canon for the second, third, and fourth figures, we have as yet no positive proof that the six moods remaining in each of those figures are valid; we merely know that they do not offend against any of the syllogistic rules. But, if we can *reduce* them, i. e. bring them back to the first figure, by shewing that they are only different statements of its moods, or, in other words, that precisely the same conclusions can be obtained from equivalent premisses in the first figure, their validity will be proved beyond question. There are two methods of performing this operation: 1st, that called *Ostensive Reduction*, which consists in employing one or more of the processes of conversion, permutation, and transposition of premisses; 2nd, that called *Reductio per impossibile*, which consists in shewing, by means of the first figure and the laws of opposition, that the contradictory of the conclusion is false, and therefore the conclusion itself true. Either of these methods is applicable to all the eighteen moods, and the result is that all are proved to be valid. I shall give instances of the application of each method.

By ostensive reduction I shall test EAO in the fourth, IAI in the third, AEE and AOO in the second figures.

Fig. 4.

E No A is B.
A All B is C.
O .·. Some C is not A.

Fig. 3.

I Some B is A.
A All B is C.
I .·. Some C is A.

Fig. 2.

A All A is B.
E No C is B.
E .·. No C is A.

Fig. 2.

A All A is B.

O Some C is not B.
O .·. Some C is not A.

Fig. 1.

.·. No B is A. (Simple Conversion.)
.·. Some C is B. (Conversion per acc.)
Some C is not A.

Fig. 1.

All B is C.
.·. Some A is B. (Simple Conversion.)
Some A is C.
.·. Some C is A. (Simple Conversion.)

Fig. 1.

.·. No B is C. (Simple Conversion.)
All A is B.
No A is C.
.·. No C is A. (Simple Conversion.)

Fig. 1.

.·. No A is not-B. (Permutation.)
.·. No not-B is A. (Simple Conversion.)
.·. Some C is not-B. (Permutation.)
Some C is not A.

The mark × shews that the premisses are transposed; the sign .·. on the right-hand side of the page is here appropriated to express the employment of conversion or permutation. The last example is interesting, because AOO in fig. 2, and OAO in fig. 3, inasmuch as they contain O premisses, cannot be reduced by the ordinary methods of transposition of premisses and conversion. Hence the older logicians (who, with few exceptions, did not recognise permutation) applied to them the tedious method of *reductio per impossibile* (or, if we write it in full, *reductio per deductionem ad impossibile*). This method is equally applicable to all the *imperfect moods*, as the moods of the three last figures are often called. I now proceed

to give an example of it, and shall select AAI in the third figure.

A All B is A,
A All B is C;
I ∴ Some C is A.

This conclusion must be true: for, if not, suppose it to be false;

Then its contradictory must be true, i. e.

No C is A.
But (from the premisses) All B is C. } Syll. II.
∴ (By figure 1) No B is A.

But (from the premisses) All B is A.

Now these two (being contrary propositions) cannot both be true.

But the proposition All B is A is assumed to be true.

∴ The proposition No B is A must be false.

Hence, either the reasoning of Syll. II. is faulty, or one of the premisses is untrue.

But the reasoning (being in the first figure) must be valid.

∴ One of the premisses is false.

Now the premiss 'All B is C,' being one of the premisses of the original syllogism, is assumed to be true.

∴ The other premiss (No C is A) must be false.

∴ Its contradictory (Some C is A) is true.

Q. E. D.

As the positive test of reduction confirms in every case the negative test of the syllogistic rules, it follows that six moods (though not the same six moods) are valid in each figure. These moods may be remembered by means of the mnemonic lines:—

*Barbara, Celarent, Darii, Ferio*que, prioris:
Cesare, Camestres, Festino, Baroko, secundæ:
Tertia, *Darapti, Disamis, Datisi, Felapton,*
Bokardo, Ferison, habet: Quarta insuper addit
Bramantip, Camenes, Dimaris, Fesapo, Fresison:
Quinque *Subalterni* totidem Generalibus orti,
Nomen habent nullum, nec, si bene colligis, usum.

In the above lines, the initial consonants, B, C, D, F, shew that the mood in the second, third, or fourth figure to which they are prefixed is to be reduced to the mood correspondingly marked in the first. Thus Disamis, when reduced, will become Darii. The vowels shew the moods; thus Disamis represents IAI in the third figure. The letter *s*, when it occurs after a vowel, shews that the proposition for which that vowel stands is to be converted simply, the letter *p* that it is to be converted *per accidens*[8].

[8] It should be noticed that when these letters follow the conclusion, as is the case in *Camestres, Disamis, Bramantip, Camenes, Dimaris,* they apply not to the conclusion of the mood given for reduction but to the conclusion of the equivalent mood in the first figure, to which such mood is to be reduced. Thus, in reducing *Bramantip,* the student will find that the *p* affects not the I conclusion of *Bramantip,* but the A conclusion of the corresponding syllogism in Barbara. See, for other instances, p. 101.

The letter *m* shews that the premisses are to be transposed, *k* that the mood is to be reduced *per impossibile*. It will be noticed that *k* occurs only in two moods, Baroko and Bokardo, but I have shewn that the *per impossibile* method is equally applicable to all imperfect moods, and that these two moods can be reduced ostensively by means of permutation, so that any imperfect mood may be reduced either ostensively or *per impossibile*[9]. The initial B in Baroko and Bokardo shews that the *per impossibile* method, in their case, assumes the validity of *Barbara*, but in other cases the operation may assume the validity of some one of the other moods in the first figure; thus, in the particular instance I have taken above, it is performed by means of *Celarent*. It is perhaps needless to add that all letters, not already explained in the mnemonic lines, are non-significant.

The nature of the *subaltern moods* has already been explained. They are, AAI, EAO in fig. 1, EAO, AEO in fig. 2, and AEO in fig. 4, included respectively in AAA, EAE, EAE, AEE, AEE. They cannot properly be regarded as illegitimate, inasmuch as the conclusions are valid, but they are superfluous, inasmuch as they infer less than is justified by the premisses.

[9] If we take κ to signify the employment of permutation, Baroko and Bokardo may be replaced respectively by Faκsoκo and Doκsamosκ. These terms are adapted from Whately's Logic, Bk. II. ch. iii. § 5.

The Special Rules.

Besides the general syllogistic rules, already enunciated and proved, certain Special Rules have been enunciated for each figure. I give them below as generally stated. Those for the first figure have been proved in establishing its canon; those for the other figures the student may verify for himself by applying the rules, already laid down, on the distribution of terms.

Fig. 1. (α) The minor premiss must be affirmative.
(β) The major premiss must be universal.

Fig. 2. (α) One or other premiss must be negative.
(β) The conclusion must be negative.
(γ) The major premiss must be universal.

Fig. 3. (α) The minor premiss must be affirmative.
(β) The conclusion must be particular.

Fig. 4. (α) When the major premiss is affirmative, the minor must be universal.
(β) When the minor premiss is affirmative, the conclusion must be particular.
(γ) In negative moods the major premiss must be universal.
(δ) The conclusion cannot be an universal affirmative, nor either of the premisses a particular negative.

Note.—If, leaving out of consideration the conclusion, we regard the number of possible figures as three and that of possible moods as sixteen, we may proceed as follows.

Having enunciated the canon of the first figure, we may constitute the moods *Barbara*, *Celarent*, *Darii*, and *Ferio*. The subaltern moods, AAI and EAO, are not admissible, as the question here before us is not 'What conclusions are justified by such and such premisses,' but 'To what conclusions do such and such premisses lead?' Now, from this point of view, the conclusions of the subaltern moods are not directly inferred from the premisses, but are inferred by subalternation from the universal conclusions to which the premisses directly lead. The same observation will of course apply to the subaltern moods in the other figures.

If we take no account of the conclusion, we have no means of determining which is the major and which is the minor term. Consequently, the premisses may lead to two kinds of conclusions: 1st, those in which the predicate of the first premiss is predicated of the subject of the second; 2nd, those in which the subject of the second premiss is predicated of the predicate of the first. Now the canon of the first figure applies only to the first case; consequently we are bound to ask if any conclusions, falling under the second head, may be inferred from the premisses. These cannot be determined directly by the canon, but must be determined in the same manner as conclusions in the second and third

figures. Here we proceed by a method similar to that employed in the text. The syllogistic rules exclude seven of the sixteen possible moods, viz. EE, EO, OE, OO, II, IO, OI. When the moods are referred to their several figures, we find that, where the extreme employed in the first premiss becomes the predicate, and the extreme employed in the second premiss the subject of the conclusion, the results are, in the second figure, *Cesare*, *Camestres*, *Festino*, *Baroko*; in the third, *Darapti*, *Disamis*, *Datisi*, *Felapton*, *Bokardo*, *Ferison*, the subaltern moods of the second figure EAO, AEO, being inadmissible. Where the extreme employed in the first premiss becomes the subject, and the extreme employed in the second premiss the predicate of the conclusion, the results are, in the first figure, AAI, AEO, AII, EAE, IEO, which, when we transpose the premisses, become respectively *Bramantip*, *Fesapo*, *Dimaris*, *Camenes*, and *Fresison* in the fourth figure. Hence, according to this mode of treatment, the moods of the fourth figure are regarded as *indirect moods* of the first. Similarly, in the second figure we may constitute the *indirect moods* AEE, EAE, IEO, OAO. These, if we transpose the premisses, are merely a repetition of the ordinary moods of the second figure. This is also the case with the indirect moods of the third figure, viz. AAI, AEO, AII, AOO, IAI, IEO. It will therefore be seen that, with the exception of rejecting the subaltern moods, which even there we regarded as superfluous, we arrive practically at the same results as in the text. The moods

of the fourth figure are recognised, but, instead of being regarded as moods of a distinct figure, they are treated as indirect moods of the first. By the expression 'indirect moods,' it will be seen, we mean moods in which the extreme employed in the first premiss becomes the subject, and the extreme employed in the second premiss the predicate of the conclusion.

CHAPTER IV.

On Trains of Reasoning. (*The Sorites.*)

SYLLOGISMS may be combined in what is called a *Train of Reasoning*. Thus the major and minor premisses, or either, of our ultimate syllogisms may themselves be proved by syllogisms; the major and minor premisses of these, or either, by other syllogisms, and so on, till at last we come to premisses not admitting of syllogistic proof. Such premisses are either assumed without any proof at all, or they are the result either of direct observation or of the testimony of others or of Induction.

In a train of reasoning, any syllogism proving a premiss of a subsequent syllogism is called with reference to the subsequent syllogism a *Pro-Syllogism*, and the subsequent syllogism with reference to it an *Epi-Syllogism*. It is obvious that the very same syllogism in different relations may be called a Pro-Syllogism or an Epi-Syllogism.

The *Sorites* (or, as it is sometimes called, the *chain-argument*) is a common instance of a train of reasoning in a compressed form. It consists, in its proper and most usual form, of a series of propositions, in which the predicate of the preceding is always the subject of the succeeding premiss. The conclusion predicates the last predicate of the first subject.

Thus,—All A is B,
All B is C,
All C is D,
All D is E;
∴ All A is E.

When expanded, the Sorites contains as many syllogisms as there are propositions intermediate between the first proposition and the conclusion. These syllogisms are in the first figure, and the conclusion of each becomes the minor premiss of the next. Thus, the above Sorites contains three syllogisms, viz.—

(1) All B is C,
All A is B;
∴ All A is C.

(2) All C is D,
All A is C;
∴ All A is D.

(3) All D is E,
All A is D;
∴ All A is E.

In a Sorites, only one premiss can be particular, viz. the first; and only one negative, viz. the last[1].

[1] The first premiss, if particular, may be stated in the form 'Some B is A,' instead of in the usual form 'Some A is B'; the first syllogism of the expanded Sorites will then be in the third figure instead of the first. Similarly, the last premiss, if negative, may be stated in the form 'No E is D,' instead of in the form 'No D is E,' which will

For, if there were a particular premiss in any place except the first, there would be a particular major, which, in the first figure, is inadmissible.

Again, if any premiss except the last were negative, there would be a negative conclusion in one of the previous syllogisms; this conclusion would necessitate, in the following syllogisms, a negative minor, which, in the first figure, is inadmissible.

Note.—Besides the above form of Sorites, there is another called the *Regressive* or *Goclenian* Sorites (so called from Goclenius, who first noticed it). Its properties are exactly the reverse of those of the ordinary Sorites. The subject of each premiss becomes the predicate of the next, the conclusion predicates the first predicate of the last subject, the conclusion of each of the expanded syllogisms becomes the major premiss of the next, and the rules by which it is governed are that the first premiss only can be negative and the last particular. It may be stated thus:—

All E is F,
All D is E,
All C is D,
All B is C,
All A is B;
∴ All A is F.

make the last syllogism of the expanded Sorites a syllogism of the second figure. No advantage, however, is gained by this mode of statement, and it is not nearly so simple as the usual form.

CHAPTER V.

On Complex (Hypothetical) Propositions and Syllogisms.

HITHERTO I have treated only of simple (or, as they have been inaccurately termed, Categorical[1]) propositions and syllogisms. A *Complex* (or, as it is commonly called, a *Hypothetical*) Syllogism is one in which one or more complex (or hypothetical) propositions occur. A complex proposition is a combination of two or more simple propositions in one sentence, the propositions being so related that the truth or falsity of one proposition or set of propositions is made to depend on the truth or falsity of the other proposition or set of propositions. If the two propositions or sets of propositions be associated together, so that the truth of one depends on the truth of the other, the complex proposition may be called *Conjunctive*[2]. If they be dissociated,

[1] The word categorical (*κατηγορικός*) properly means affirmative, and is always used in that sense by Aristotle.

[2] I have, in accordance with more ancient usage, employed the word 'conjunctive' in place of the word 'conditional,' which is employed by Aldrich and other logicians of his time. Instead of simple and complex propositions, they speak of categorical and hypothetical, subdividing hypothetical into conditional and disjunctive. Besides the improper use of the word categorical (noticed

so that the truth of one depends on the falsity of the other, and the falsity of one on the truth of the other, the complex proposition may be called *Disjunctive*. Thus we may take as instances of Conjunctive Propositions :—

If (or When, Where, Provided that, &c.) A is B, C is D; If A is not B, C is D; If A is not B, C is not D; If A is B and C is D, E is F; If A is B, either C is D or E is F or G is H; If either A is B or C is D, E is F.

It will be noticed in the second and third examples that negatives are introduced, but they are, notwithstanding, examples of conjunctive propositions, for C being D in the first case, and C not being D in the second, are made to depend on the *truth* of A not being B. As instances of Disjunctive Propositions we may take the following :—

Either A is B, or C is D; Either A is B, or C is D, or E is F; A is either B or C or D; Either A or B or C is D; Either A is not B, or C is not D; Either A is B, or C is not D.

This form of proposition implies that the truth of one member involves the falsity of the other, and, vice versâ, the falsity of one member the truth of the other.

It should be noticed that both conjunctive and disjunctive propositions admit of being reduced to the simple form. Thus :—

above), it is extremely awkward to make hypothetical and conditional (which are synonyms) stand respectively for the genus and species. The words conjunctive and disjunctive serve also to point out that the division of complex propositions is exhaustive.

'If A is B, C is D' becomes 'The case of A being B is a case of C being D' or 'A being B involves as a consequence C being D.'

The disjunctive proposition, when analysed, contains four conjunctive propositions, each of which may be reduced to a simple proposition. Thus, 'Either A is B, or C is D' is equivalent to the four conjunctive propositions: If A is B, C is not D; If A is not B, C is D; If C is D, A is not B; If C is not D, A is B. Of these four propositions, however, the third is implied in the first, and the fourth in the second.

I now proceed to consider Complex Syllogisms, i.e. syllogisms which contain Complex Propositions.

§ 2. I. *Conjunctive Syllogisms.*

A Conjunctive Syllogism is a syllogism, one or both of whose premisses are conjunctive propositions; if only one premiss be conjunctive, the other must be simple. If both premisses be conjunctive, inasmuch as all conjunctive propositions rank as universal affirmatives, the syllogism, to be valid, must be conformed to Barbara in the first figure. Thus,

If A is B, C is D,
If C is D, E is F;
∴ If A is B, E is F,

is a vallid syllogism; but the following would not be valid:

If A is B, C is D,
If A is B, E is F;
∴ If C is D, E is F.

Far the most common form however of a conjunctive syllogism is that in which the major is a conjunctive, and the minor a simple proposition. Of this form there are four possible varieties, of which two are valid and two invalid. These may be represented thus:—

If A is B, C is D. (Major premiss.)

(1) A is B;	(β) C is D;
∴ C is D.	No conclusion.
(α) A is not B;	(2) C is not D;
No conclusion.	∴ A is not B.

Hence we obtain the rule that, if we affirm the antecedent, we must affirm the consequent, or, if we deny the consequent, we must deny the antecedent; but, if we deny the antecedent or affirm the consequent, no conclusion can be drawn. The reason of this rule will be obvious on a little reflexion. We assert that 'A being B involves as a consequence C being D'; hence, if we grant that A is B, it must follow that C is D; if we deny that C is D, it must follow that what involves it as a consequence must also be untrue; but C might still be D, though A were not B, nor would it follow from C being D that A was also B.

Syllogism (1) is called a *Constructive* conjunctive syllogism.

Syllogism (2) is called a *Destructive* conjunctive syllogism.

It may be useful to add a few examples of valid conjunctive syllogisms.

(1) If A is B, C is not D.

C is D;

∴ A is not B.

(2) If A is not B, C is D.

A is not B; | C is not D;

∴ C is D. | ∴ A is B.

(3) If A is not B, C is not D.

A is not B; | C is D;

∴ C is not D. | ∴ A is B.

(4) If A is B, either C is D or F is G.

A is B; | Neither C is D nor F is G;

∴ Either C is D or F is G. | ∴ A is not B.

(5) If either C is D or F is G, either X is Y or V is W.

Either C is D or F is G; | Neither X is Y nor V is W;

∴ Either X is Y or V is W. | ∴ Neither C is D nor F is G.

§ 3. II. *Disjunctive Syllogisms.*

A Disjunctive Syllogism is a syllogism of which the major premiss is a disjunctive, and the minor a simple proposition, the latter affirming or denying one of the alternatives stated in the former.

We may indeed combine two disjunctive propositions, and draw conclusions from them, but we can only do so after reducing the disjunctive propositions to the conjunctive form. Thus from the two propositions Either

A is B or C is D, Either A is B or E is F, we may draw four conclusions, viz. If C is D, E is F; If C is not D, E is not F; If E is F, C is D; If E is not F, C is not D. But, as these conclusions are really drawn from conjunctive propositions which are involved in the two disjunctive propositions, we are not justified in calling the syllogisms disjunctive. Hence, as will be noticed, my definition of disjunctive is not so wide as that of conjunctive syllogisms.

The disjunctive syllogism admits of four conclusions, which may be exhibited thus:—

Either A is B, or C is D.

(1) A is B; ∴ C is not D.	(3) C is D; ∴ A is not B.
(2) A is not B; ∴ C is D.	(4) C is not D; ∴ A is B.

I add a few examples:—

Either A is B, or C is not D.

(1) A is B; ∴ C is D.	(3) C is not D; ∴ A is not B.
(2) A is not B; ∴ C is not D.	(4) C is D. ∴ A is B.

Either A is B, or C is D, or E is F.

(1) A is B; ∴ Neither C is D nor E is F.

(2) A is not B; ∴ Either C is D or E is F.

(3) Neither C is D nor E is F; ∴ A is B.

(4) Either C is D or E is F; ∴ A is not B.
(5) Either A is B or C is D; ∴ E is not F.
&c. &c.

He is either a fool or a knave.

(1) He is a fool;	(3) He is not a fool;
∴ He is not a knave.	∴ He is a knave.
(2) He is a knave;	(4) He is not a knave;
∴ He is not a fool.	∴ He is a fool.

Note.—Mr. Mill (in his *Examination of Sir W. Hamilton's Philosophy*, ch. xxiii.) maintains that a disjunctive proposition merely implies that the two alternatives cannot both be false, but that it does not exclude the possibility of both of them being true. Thus, in the last example, he would maintain that there is nothing in the form of the assertion to exclude the supposition of the man being both a fool and a knave. In this opinion he is preceded by many other logicians, but it seems to me that in the expression 'either —— or ——' we distinctly exclude the possibility of both alternatives being true, as well as of both being false. In fact, when we do not wish to exclude the possibility of both being true, we add the words 'or both,' thus: 'He is either a fool or a knave, or both'; 'I shall come either to-day or to-morrow, or perhaps both days.'

§ 4. *The Dilemma.*

There remains the case in which one premiss of the complex syllogism is a conjunctive and the other a disjunctive proposition, it being, of course, understood that the disjunctive proposition deals only with expressions which have already occurred in the conjunctive proposition. This form is called a *Dilemma.* The order of the premisses is different, but it seems more natural that the conjunctive proposition should be the major. If we consider the case in which the major consists of one antecedent and several consequents, there is only one valid form of argument, and that is destructive.

(1) If A is B, C is D and E is F;
But either C is not D or E is not F;
∴ A is not B.

If we asserted in the minor 'C is D and E is F' there would be no conclusion, and if we asserted 'Neither C is D nor E is F,' the minor would not be disjunctive. The assertion 'Either C is D or E is F' is, according to my view of the significance of a disjunctive proposition, equivalent to the assertion 'Either C is not D or E is not F,' and leads to the same conclusion.

If the major consist of several antecedents, and one consequent, there is only one valid form of argument, and that is constructive.

(2) If A is B or if E is F, C is D;
But either A is B or E is F;
∴ C is D.

If we asserted in the minor 'C is not D,' it would not satisfy the requirements of the definition by being a disjunctive proposition.

In the remaining case, where there are several antecedents and several consequents, there are two valid forms, one constructive and the other destructive.

(3) If A is B, C is D; and if E is F, G is H;
But either A is B, or E is F;
∴ Either C is D, or G is H.

(4) If A is B, C is D; and if E is F, G is H;
But either C is not D, or G is not H;
∴ Either A is not B, or E is not F.

It is evident that we may form a Trilemma, Tetralemma, &c., by increasing the number of antecedents or consequents or both, thus:—

If A is B, or if E is F, or if G is H, C is D;
But either A is B, or E is F, or G is H;
∴ C is D.

If A is B, C is D; and if E is F, G is H; and if I is J, K is L;
But either A is B, or E is F, or I is J;
∴ Either C is D, or G is H, or K is L.

It is not uncommon to mistake for dilemma what is really only a conjunctive syllogism. Thus the two following syllogisms, when examined, will be found to be, the first a constructive, the second a destructive conjunctive.

(1) Whether geometry be regarded as a mental discipline or as a practical science, it deserves to be studied;

But geometry may be regarded as both a mental discipline and a practical science;

∴ It deserves to be studied.

(2) If we go to war, we must either contract a debt, or increase the taxation, or indemnify ourselves at the enemy's expense;

We shall not be able to do any of these;

∴ We are not able to go to war.

In disputation, the adversary who is refuted by a dilemma is said to be 'fixed on the horns of a dilemma'; he is said to *rebut* the dilemma, if he meet it by another with an opposite conclusion. Thus (to tell an old story) Protagoras the Sophist is said to have engaged with his pupil, Euathlus, that half the fee for instruction should be paid down at once, and the other half remain due till Euathlus should win his first cause. Euathlus deferred his appearance as an advocate, till Protagoras became impatient and brought him into court. The Sophist then addressed his pupil as follows: 'Most foolish young man, whatever be the decision, you must pay your money; if the judges decide in my favour, I gain my fee by the decision of the court, if in yours by our bargain.' This dilemma Euathlus *rebutted* by the following: 'Most sapient master, whatever be the decision, you must lose your fee; if the judges decide in my favour, you lose it

by the decision of the court; if in yours, by our bargain, for I shall not have gained my cause.'

Note 1.—Of the four cases of dilemma which I have given, the first would not be admitted by Abp. Whately and Dr. Mansel, who define dilemma as 'A syllogism having a conditional (i. e. conjunctive) major premiss with more than one antecedent, and a disjunctive minor.' Having however a disjunctive minor, it cannot properly be regarded as a conjunctive syllogism, and it seems less arbitrary and more systematic to define dilemma as 'a syllogism of which one premiss is a conjunctive and the other a disjunctive proposition' than to limit it as above.

Note 2.—Few parts of Logic have occasioned more differences of opinion or nomenclature than the theory of complex (or hypothetical) propositions and syllogisms. Sir W. Hamilton (see his *Lectures on Logic*, Lecture xiii. and Appendix viii.) finally arrives at the opinion that 'hypothetical and disjunctive judgments' are not more complex than ordinary propositions, and that 'hypothetical and disjunctive reasonings' are really forms of immediate inference. Thus he would represent the conjunctive syllogism in the form:—

If A is B, C is D;
∴ A being B, C is D.

The disjunctive syllogism he would represent in the form:—

Either A is B, or C is D;
∴ A not being B, C is D.

The other inferences from the premisses would, of course, be drawn similarly.

The dilemma would assume the form:—

If A is B, C is D; and if E is F, G is H;

∴ Either A being B, or E being F, it follows that C is D, or G is H;

or ∴ Either C not being D, or G not being H, it follows that either A is not B, or E is not F.

Without attempting to discuss the arguments that have been adduced on either side, I may express my own opinion that complex (or hypothetical) propositions and syllogisms are rightly so called, and that the latter are to be regarded as mediate, and not as immediate, forms of inference. Though no new term is introduced in the minor premiss, the major and minor premisses are entirely distinct propositions, and the conclusion is the result not of one proposition but of two propositions taken conjointly.

CHAPTER VI.

On the words 'Most,' 'Many,' &c., as expressing the Quantity of Propositions.

TO all particular propositions I have hitherto prefixed the word 'some.' Both in conversation and reasoning, however, it frequently happens that we use some other sign of particularity, such as 'many,' 'most,' &c. Nor does there seem any valid reason why these forms should not be recognised by Logic. From what has already been said of particular premisses, it will be seen that wherever one premiss is universal and the other modified by some sign of particularity, as 'some,' 'many,' 'most,' &c., the conclusion must be particular, the degree of particularity in no case transcending (though, in some cases, it may fall below) that denoted by the particular premiss. Thus, if we state as our premisses 'All crimes are to be punished,' 'Many offences against individual persons are crimes,' we must draw the conclusion that 'Many offences against individual persons are to be punished.' But if we state as our premisses 'All crimes are to be punished,' 'Most crimes are offences against individuals,' we are only justified in drawing the conclusion that 'Some offences against

individuals are to be punished.' The student will, from the principles already laid down, be easily able to distinguish between the two classes of cases.

'Two particular premisses prove nothing.' This is a general rule, and is strictly true where the premisses are quantified as 'some'——'some.' But there is one case in which two particular premisses necessitate a conclusion. I will begin with a simple instance of it. If two different predicates can both be predicated affirmatively of the greater number of individuals denoted by the same common term, there must be some individuals of which they can both be predicated, i. e. in certain cases the predicates must be predicable of each other. Thus, from the premisses

Most A are B,
Most A are C,

we must necessarily infer that Some A is both B and C, and consequently that Some B are C and Some C are B.

But we may draw the same conclusion, even in those cases in which both premisses are not quantified by the word 'most,' provided that the sum of the quantities by which the subjects are affected exceeds unity. Thus from the premisses

Three-fourths of A are B,
One-third of A is C,

it follows that at least one-twelfth of A is both B and C; but if B and C be both predicable of the same objects, either must be, partially, predicable of the other. If, for instance, three men out of four exceed a certain height

and one out of three a certain weight, at least one out of twelve must exceed both the given height and the given weight, and we may affirm both that Some men who exceed a certain height also exceed a certain weight, and that Some men who exceed a certain weight also exceed a certain height.

Of course, when we use such an indefinite word as 'most' in either premiss, the other premiss must be quantified by an expression signifying at least one-half; else we cannot be sure that the quantities of the two premisses, when added together, exceed unity.

A conclusion of this kind can only be drawn where the subject in both premisses is the same term, i.e. in the third figure; for, in the mere form of a logical proposition, we have no data to guide us with regard to the quantity of the predicate. Thus, from the premisses

Nineteen-twentieths of A are B,
Nine-tenths of B are C,

we can draw no conclusion as to the relation of A and C; for the tenth of B which is not C might be precisely that portion which was coincident with, or which contained, the nineteen-twentieths of A. Though, however, these syllogisms are confined to the third figure, they may be either affirmative or negative. Thus, from the premisses

Three-fourths of A are not B,
Two-thirds of A are C,

we may infer that five-twelfths at least of A are C and not B, and consequently that some C is not B, and some things which are not B are C.

Note.—The propositions employed in the above chapter have usually been regarded as Particulars, though they have sometimes been classed with Universals. See Sir W. Hamilton's *Lectures on Logic*, vol. ii., first ed. p. 354; second ed. p. 361.

CHAPTER VII.

On Probable Reasoning.

IN discussing the copula, it was maintained that any modification of our assertions, such as the qualifications introduced by the words 'probably,' 'possibly,' &c., was, in the ultimate analysis of the proposition, to be referred to the predicate and not to the copula. Thus such a proposition as 'A is probably B,' when stated in its strictly logical form, would become 'That A is B *is* a probability.' It would however be tedious and practically useless to reduce all our propositions to such a form. I may therefore proceed to lay down rules for reasoning from propositions whose copula is modified, remembering however that they are not stated in strictly logical language.

The correctness of the following rule will be apparent. If a premiss whose copula is modified be combined with another premiss whose copula is unmodified, the copula of the conclusion must be modified also; the modality, of course, never transcending that of the premiss. Thus from the premisses 'All true poets are men of genius,' 'Sophocles is probably (certainly, possibly, &c.) a true poet,' I infer that Sophocles is probably (certainly, possibly, &c.) a man of genius. From a certain and a

probable premiss, therefore, arranged according to the ordinary laws of syllogism, we can never infer more than a probable conclusion. To this head may most conveniently be referred those syllogisms in which the major is a particular proposition introduced by the word 'most,' and the minor a singular proposition. Thus from the premisses, 'Most philosophers are men of vivid imagination,' 'A B is a philosopher,' I infer, as the conclusion, A B is probably a man of vivid imagination. If *most* philosophers possess certain characteristics, any particular philosopher will *probably* possess them, so that the major premiss is, in fact, equivalent to the proposition, 'A philosopher is probably a man of vivid imagination.'

Using the word 'probable' in the sense of 'more likely than not,' two probable premisses do not lead to a probable conclusion. This fact will be obvious from an easy example. Suppose there are in a bag four red, five blue, and six white balls: I may say with truth 'Any ball drawn at random from the bag is probably a red or blue ball'; I may also say with truth 'Any ball drawn at random from the red and blue balls is probably a blue ball'; but I cannot infer that 'Any ball drawn at random from the bag is probably a blue ball.' I shall only be justified in drawing the conclusion 'Any ball drawn at random from the bag is *possibly* a blue ball.' But, where our information is so special as in the above instance, a conclusion of this kind is far too vague. Is there no method which will enable me to state in the conclusion the exact value of the expectation that any

particular ball drawn at random may be blue, red, or white? For such a method I must have recourse to mathematics.

Though the word 'probable' is used in the sense of 'more likely than not,' the word *probability* is used as the equivalent of 'chance' or 'expectation.' If it be three to two that a certain event will happen, 3 : 2 is called the *odds* for, 2 : 3 the *odds* against the event. Now the 'probability' or 'chance' of the event happening would be expressed by $\frac{3}{5}$, that of its not happening by $\frac{2}{5}$, the denominator in both cases being expressed by the sum of the terms of the odds, the numerator in the first case by the term of the odds for, in the latter case by the term of the odds against. If two events are independent of each other, the joint or compound probability that they will both happen must be much smaller than the probability that either of them will happen alone, and it is discovered by multiplying together the fractions which express the probabilities of their happening separately[1]. Thus, in the above instance, the chance of my drawing a red or blue ball = $\frac{9}{15}$; the chance of my drawing out of the red and blue balls a blue ball = $\frac{5}{9}$; ∴ the chance of my drawing a blue ball out of a bag which contains all = $\frac{9}{15} \times \frac{5}{9} = \frac{1}{3}$, a result at which, in this particular instance, I could of course have arrived more directly. Hence, when both premisses are affected by words like 'probably,' 'possibly,' &c., the probability of the conclusion

[1] The truth of this proposition, with regard to two, three, or any number of events, is proved at length in Peacock's *Arithmetical Algebra*, § 469.

may always be discovered by multiplying together the probabilities of the premisses, the conclusion being therefore less probable than either premiss. I append a few instances of conclusions drawn from probable premisses:—

(1) This plant will probably sprout up during the winter months. (Let the probability = $\frac{3}{5}$.)

Whatever plant sprouts up during the winter will probably be bitten by the frost. (Let the probability = $\frac{4}{7}$.)

∴ This plant may sprout up and be frost-bitten. (Here the probability = $\frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$.)

∴ The odds *against* the event are 23 to 12, and those in *favour* of it 12 to 23.

(2) Two thirds of these men will be enlisted.

Half the men enlisted are killed in battle.

∴ The probability of any particular man being enlisted and killed in battle = $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$.

∴ It is two to one against any particular man here being enlisted and killed in battle.

(3) A warm day may possibly be a rainy day. (Let the probability = $\frac{1}{5}$.)

A rainy day is probably a calm day. (Let the probability = $\frac{4}{5}$.)

∴ A warm day may possibly be both rainy and calm[2]. (The probability = $\frac{4}{15}$.)

[2] I am indebted to Professor Shaw, of Derry, for pointing out an inaccuracy in the conclusion of this example, as stated in the earlier editions, and to Professor Park for the suggestion that it is a peculiarity of all arguments of this kind that, strictly speaking, both premisses should be expressed in the conclusion.

N. B.—It is most important for the student to bear in mind the ambiguous use of the words 'probably,' 'probable,' 'probability.' The adverb 'probably' we seem always to use in the sense of 'more likely than not.' The adjective 'probable,' when employed as a predicate, seems also to be invariably used in the same sense; thus we say 'It is probable that he will do so and so,' 'This event is probable.' But when used to qualify a substantive, as in such expressions 'probable premisses,' 'probable reasonings,' &c., it may be employed either in the above sense or simply as contrasted with certainty, and in the latter case the 'probability,' as we say, may vary from certainty to zero. Lastly, the word 'probability' may simply be equivalent to 'chance,' as explained above, or in some expressions it may have the meaning of 'being more likely than not,' as when we say 'The probability is that he will do so and so.' By 'probable reasoning' at the head of this section I of course mean reasoning which falls short of certainty, and the value of which may vary to any extent so long as it does not rise to certainty or fall to zero.

On the Combination of Probable Arguments.

[*Cumulative or Corroborative Evidence, including Circumstantial Evidence.*]

Probable arguments may be combined together in a chain[3] (or, as it has been more appropriately called, a coil) of reasoning, each argument leading to the same conclusion. Instead of weakening each other, as is the case with probable premisses, such arguments, being all independent testimonies to the truth of the same conclusion, mutually strengthen each other. If the value of any single argument amounts to certainty, the conclusion must be true. In this case therefore we have to calculate the chances of failure in each separate argument; these, when multiplied together, give the probability of all the arguments together failing to prove the conclusion; and this fraction, when subtracted from unity (which represents certainty), gives the probability, resulting from all the arguments jointly, in favour of the conclusion. Thus suppose the probabilities in favour of certain probable arguments to be represented respectively by $\frac{1}{3}$, $\frac{4}{5}$, $\frac{3}{4}$;

[3] Chain-reasoning would be the proper designation for a series of arguments (or links of evidence) which are all inter-dependent, so that, if one argument fails (or one link snaps), the reasoning breaks down altogether. Much legal evidence is of this kind, as when A testifies that he received a certain parcel from B, B that he received it from C, and so on. In this case, any one untrustworthy witness would invalidate the whole evidence. On the other hand, in corroborative evidence the various parts strengthen one another.

the chances of their failing to prove the conclusion will be represented respectively by $\frac{2}{3}$, $\frac{1}{5}$, $\frac{1}{4}$ (or the differences between the favourable chances and unity); the chance therefore of their all failing to prove the conclusion $= \frac{2}{3} \times \frac{1}{5} \times \frac{1}{4} = \frac{1}{30}$; consequently the probability in favour of the conclusion, as based upon all the arguments jointly, is $\frac{1}{30}$, i. e. the odds in favour of it are 29 to 1.

I may illustrate this case by an example, which will also serve to shew the characteristic uncertainty attaching to this method of reasoning. Suppose a man to be found lying dead on a road from the effects of a wound. On the same evening on which he died, another man was seen running away from the neighbourhood of the place. On this man's house being searched, his clothes are found to be stained with blood; his footsteps correspond with those leading to and from the place where the dead man was lying; and moreover he is known to have possessed a weapon, now not to be found, which was capable of inflicting the wounds. The presumption in favour of his guilt is very great; each argument, taken alone, possesses some cogency, and when all the arguments are taken together they appear to be irresistible. But suppose the suspected man, when arrested, to give this account of the affair: he was walking along the road, armed with a dagger; he was suddenly attacked by another man; a scuffle ensued, and in the scuffle he killed his assailant; finding that he had killed him, he was seized with a sudden panic, threw away his weapon, and ran home. Such an account, in the case

of a timid and secretive man, might possibly be true, and, in estimating the counter-probabilities, we should have to consider the characters of the accused and the dead man, and the nature of the motive, if any, which could have led to the supposed crime. Suppose the dead man's pockets were rifled, and the accused (who had been previously convicted of a felony) were in possession of his money, there could be little doubt that he had committed a murder; but suppose that the character of the accused was good, and no likely motive could be assigned for the commission of the crime, his own version of the affair might be accepted as probably true, or at least as throwing considerable doubt on the supposition of his guilt[4].

[4] As illustrating the danger of exaggerating the value to be attached to this kind of evidence, I could hardly adduce a more forcible example than the following passage from Lord Coke (quoted by Bentham in his *Rationale of Judicial Evidence*, Bk. V. ch. xv. § 2):—

'Violenta presumptio is many times plena probatio; as if one be run thorow the bodie with a sword in a house, whereof he instantly dieth, and a man is seen to come out of that house with a bloody sword, and no other man was at that time in the house.'

To this Bentham replies by two counter-suppositions:—

'1. The deceased plunged the sword into his own body, as in the case of suicide; the accused, not being in time to prevent him, drew out the sword, and so ran out, through confusion of mind, for chirurgical assistance.

'2. The deceased and the accused both wore swords. The deceased, in a fit of passion, attacked the accused. The accused, being close to the wall, had no retreat, and had just time enough to draw his sword, in the hope of keeping off the deceased; the deceased, not seeing the sword in time, ran upon it, and so was killed.

'Other suppositions might be started besides these; nor do these

The commonest example of evidence of this kind (which may be called Cumulative or Corroborative Evidence) is *Circumstantial Evidence*, so called from the fact that

exculpative ones either of them seem in any considerable degree less probable than that criminative one: if so, the probability of delinquency, instead of being conclusive, is but as 1 to 2.'

Sometimes the individual arguments in evidence of this kind are of so little value that, even when several of them are accumulated, they have no practical force. 'Presumptio probabilis,' Lord Coke says rightly, 'moveth but little, but presumptio levis seu temeraria moveth not at all.'

On the other hand, the force of this evidence may amount to that of moral certainty. 'If,' says Mr. Wills, 'it be proved that a party charged with crime has been placed in circumstances which commonly operate as inducements to commit the act in question,—that he has so far yielded to the operation of those inducements as to have manifested the disposition to commit the particular crime,—that he has possessed the requisite means and opportunities of effecting the object of his wishes,—that recently after the commission of the act he has become possessed of the fruits or other consequential advantages of the crime,—if he be identified with the *corpus delicti* by any conclusive mechanical circumstances, as by the impressions of his footsteps, or the discovery of any article of his apparel or property at or near the scene of the crime,—if there be relevant appearances of suspicion, connected with his conduct, person, or dress, and such as he might reasonably be presumed to be able, if innocent, to account for, but which, nevertheless, he cannot or will not explain,—if, being put upon his defence *recently* after the crime, under strong circumstances of adverse presumption, he cannot shew where he was at the time of its commission,—if he attempt to evade the force of those circumstances of presumption by false or incredible pretences, or by endeavours to evade or pervert the course of justice,—the concurrence of all or many of these cogent circumstances, inconsistent with the supposition of his innocence and unopposed by facts leading to a counter-presumption, naturally, reasonably, and satisfactorily establishes the moral certainty of his

several circumstances are adduced as all supporting the same conclusion. It will be readily seen that the utmost caution is required in estimating its value. We are bound to consider not only the circumstances which point to the conclusion, but also those which make against it, or in favour of any counter-supposition. It is only when we feel certain that we have exhausted all possible suppositions, consistent with the circumstances of the case, and considered carefully the value of the arguments, or series of arguments, pointing to each of them, that we are entitled to pronounce with confidence in favour of any particular conclusion.

Any single syllogism in a coil of circumstantial evidence may be represented as an argument with one probable and one certain premiss, thus:

> A man who was seen running away, in order to escape observation, from the place where the dead man was lying, immediately after his death, was probably the murderer,
>
> This man was seen running away, &c.;
>
> ∴ He was probably the murderer.

The major premiss of each of these syllogisms is the result of an analogical argument; and its value must be estimated according to the rules of analogy, already explained.

guilt,—if not with the same kind of assurance as if he had been seen to commit the deed, at least with all the assurance which the nature of the case and the vast majority of human actions admit.' Wills on *Circumstantial Evidence*, 4th ed., pp. 276, 7.

Note 1.—It is more usual to regard the separate syllogisms in a coil of circumstantial evidence as syllogisms in the second figure, involving an undistributed middle, and justifying a probable conclusion. Thus the above syllogism would be represented in the form:

The murderer would, to escape observation, run away, &c.,

This man, to escape observation, ran away, &c.;

∴ This man is probably (or possibly) the murderer.

It seems however extremely awkward to represent such reasonings as fallacies, and then, by way of compensation, to regard them as probable arguments. This mode of treatment, no doubt, originated in the desire to conform the arguments in circumstantial evidence to those Enthymematic syllogisms of Aristotle which employ the σημεῖον in the second figure. For an account of the Enthymeme and its subdivisions, the student is referred to Trendelenburg's *Elements*, § 37, and Dr. Mansel's *Appendix to Aldrich*, Note F.

Note 2.—There are some cases, as, for instance, those in which the witnesses may be suspected of lying, or in which they are actuated by violent passions, where circumstantial evidence may be of far greater value than direct evidence. See some remarks on this subject in an article contributed to the *Fortnightly Review* for January, 1873, by Sir H. Maine, and since reprinted at the end of *Village Communities*. On circumstantial evidence, generally, the student would do well to consult Sir Fitzjames Stephen's *Introduction to the Indian Evidence Act*, Wills

on *Circumstantial Evidence*[5], and Taylor's *Treatise on the Law of Evidence*, Pt. I. ch. 4. In Direct Evidence, the witness testifies to the precise fact alleged; in Indirect or Circumstantial Evidence to some fact or facts indicative of the fact alleged. The rules for estimating the logical value of circumstantial evidence apply also to direct evidence in all those cases in which we have any reason for questioning either the veracity or the competence of the principal witness, and in which, therefore, his evidence requires corroboration from independent testimony.

[5] The work of Mr. Wills is exceedingly interesting and instructive; but the reader must be on his guard against an erroneous statement on the mathematical value of a coil of circumstantial evidence.

CHAPTER VIII.

On Fallacies.

§ 1. A FALLACY is, strictly speaking, a defective inference, but the word is, by common usage, extended to any error either in the premisses or in the conclusions of our arguments. In deductive Logic (for we are not here concerned with the fallacies incident to induction[1]

[1] The fallacy of False Analogy (which consists either in over-estimating, in some particular case, the value of the argument from analogy, or in supposing an analogy where none exists) falls properly within the domain of Inductive Logic, and is discussed in the author's 'Elements of Inductive Logic,' ch. vi. It is not to be confounded with the fallacy arising from the employment in a syllogism of a word used analogously, as if it were used univocally, which, as already noticed, is one case of the fallacy of ambiguous terms. Thus to argue, because there are certain points of resemblance between the development of the individual and the development of the race, that, therefore, since the individual dies, the race will probably die also, or, because there are certain points of resemblance between the earth and the other planets, that, therefore, the other planets are certainly, or very probably, inhabited, would both be instances of false analogy, the former being the assumption of an analogy which appears to have no existence, the latter being an exaggeration of the value, in that particular case, of the probable argument. But to argue, because art (i.e. artistic skill) requires the highest intellectual gifts, and dissimulation is art (i.e. deceit), that, therefore, dissimulation requires the highest intellectual gifts, is obviously a mere play upon words, and owes its semblance of reasoning simply to the ambiguity of language.

or to the operation subsidiary to it, observation) these errors are traceable to one of four sources: the assumption of a false premiss, neglect of the laws of deductive inference, irrelevancy, and ambiguity of language. Any conclusions, therefore, or series of conclusions, which transgress no law of inference, which are derived from true premisses, which are relevant to the matter under discussion, and which, with their premisses, are expressed in unambiguous language, may be regarded as faultless.

§ 2. I. A false premiss, borrowed from some science which is not under investigation, can only be detected by a special knowledge of the science from which it is taken. Many fallacies, described in the old books on Logic, are really instances of the assumption of a false premiss, and therefore specially concern other sciences rather than logic. Thus the celebrated fallacy of Achilles and the tortoise (see p. 175) rests upon a false assumption, viz. that when the distances between the two become infinitesimal they will be traversed by Achilles in finite and equal (and not, as will actually be the case, in infinitesimal and rapidly diminishing) times.

The phrase 'False Analogy' is also applied to a perversion of the Analogy of Aristotle (see p. 72); namely, to those cases in which the conclusion is based upon relations wherein the two sets of terms compared do not agree. Thus, though, in many respects, the relation of a king to his subjects is the same as that of a father to his children, it is plain that invalid conclusions might be based on this general relation, by extending it to points in which it does not subsist. Such invalid conclusions would be called 'False Analogies.'

§ 3. II. The fallacies due to the neglect of the laws of deductive inference (which, strictly speaking, are the only fallacies to be detected by a mere knowledge of Deductive Logic) have already, to some extent, been discussed. The principal sources of fallacy in a single inference are illicit process and undistributed middle.

Undistributed middle often occurs in the following form:

All Conservatives (or Liberals, Roman Catholics, Protestants, Englishmen, Frenchmen, &c.) hold such and such opinions, do such and such things, or possess such and such characteristics;

A B holds such and such opinions, does such and such things, or possesses such and such characteristics;

∴ A B is a Conservative (Liberal, Roman Catholic, &c.).

We might, of course, argue quite as legitimately that, because both men and cats are animals, all men are cats. If, however, the argument assumed this shape,—

None but Liberals (Roman Catholics, &c.) hold such and such opinions, or do such and such things;

This man holds such and such opinions, or does such and such things;

∴ He is a Liberal (Roman Catholic, &c.),

it would be perfectly legitimate.

The major premiss in this case (see p. 83) is equivalent to the proposition 'All men who hold such and such opinions, or do such and such things, are Liberals (Roman Catholics, &c.).' As was shewn in the Chapter on Proba-

bilities, we might also advance a perfectly legitimate argument of the following kind: 'A man who holds such and such opinions, or does such and such things, is probably a Liberal (Roman Catholic, &c.); this man holds such and such opinions, or does such and such things; therefore he is probably a Liberal (Roman Catholic, &c.).'

Of Illicit Process the following syllogisms may serve as examples:

No form of democracy excludes the great mass of the people from political power,
Any form of government which excludes the great mass of the people from political power is subject to violent revolutions;
∴ No form of democracy is subject to violent revolutions.

The early history of some nations is full of incredible events,
A history which is full of incredible events is not worthy of serious study;
∴ The early history of any nation is not worthy of serious study.

The former syllogism (AEE in the first figure) involves illicit process of the major, the latter (EIE in the first figure) illicit process of the minor. It will be observed that the various propositions are not stated in strictly logical form, though they easily admit of being so stated, and the premisses of both syllogisms require to be transposed.

Where there is a long train of reasoning in which one

syllogism is employed to prove the premiss of another, and so on, a fallacy frequently occurs, which often escapes detection. This is called the *Argument in a Circle.* A book is written, or a speech is made, with the object of proving some controverted opinion. The author or speaker, being full of one idea, after a little preliminary matter, assumes the proposition to be proved, slightly disguised, probably, under some equivalent form; from this proposition he deduces various conclusions, and these conclusions, when put together, of course triumphantly establish, from various sides, his view of the controversy. This is not an unfair analysis of many elaborate arguments, the fallacy being, in the great majority of instances, undesigned, and imposing on the author or speaker himself quite as much as on his readers or hearers. The fallacy, when expressed in its naked form, may be described as the assumption, in a train of reasoning, of the conclusion of a subsequent syllogism as a premiss of a precedent syllogism. It may be represented thus:—

Syll. (1)	B is A,	Syll. (2)	C is A,
	C is B;		B is C;
	∴ C is A.		∴ B is A.

Thus, when asked why B is A, we reply, because C is A; and when asked why C is A, we reply, because B is A.

Of course, in actual argument, hundreds of intermediate syllogisms might occur between syll. (1) and syll. (2). The larger the number of intermediate steps, the more likely is the fallacy to escape detection, and, conversely,

the true mode of exposing the fallacy, as Whately observes, is to *narrow* the circle by cutting out the intermediate steps, and exhibiting the assumption of the conclusion in its naked form. The fallacy is to be regarded as a breach of the laws of inference, because, when reduced to its simplest terms, it is a proving of the conclusion by means of itself, instead of by means of premisses which *jointly* necessitate it. But, unlike the fallacies of illicit process, &c., already treated, it cannot be detected from the inspection of a single syllogism, but requires the comparison of two syllogisms or more.

The argument in a circle is the most important case of the fallacy called *Petitio Principii* (or, as it is more properly called, *Petitio Quæsiti, begging the question*). The other cases of petitio principii generally enumerated, though the enumeration is by no means exhaustive, are (2) when the conclusion simply re-asserts a premiss of the same syllogism[2], (3) when it is exactly synonymous with[3]

[2] If a syllogism, in which one of the simpler forms of petitio principii occurs, were stated at length, one of the premisses would be otiose (i.e. would not contribute towards the conclusion), but, as already noticed, in our reasonings, as actually expressed, we generally suppress one of the premisses.

[3] 'The English language,' says Archbishop Whately (*Elements of Logic*, Bk. III. § 13), 'is perhaps the more suitable for the fallacy of *petitio principii*, from its being formed from two distinct languages, and thus abounding in synonymous expressions which have no resemblance in sound and no connexion in etymology; so that a sophist may bring forward a proposition expressed in words of Saxon origin, and give as a reason for it the very same proposition stated in words of Norman origin; *e.g.* "to allow every man an unbounded

one of the premisses, (4) when one of the premisses is equally unknown with the conclusion, (5) when it is more unknown. Cases (4) and (5) are really instances of the assumption of a false premiss, or at least of a premiss which is not known to be true. Cases (2) and (3) arise from a neglect of the laws of syllogism, for, instead of proving the conclusion from the two premisses *jointly*, we simply re-assert one of the premisses [4].

Connected with this fallacy is the rhetorical device of *question-begging epithets*. Thus, though the matter we are discussing is open to dispute, we may speak of a nefarious project, a laudable ambition, an astute act, a far-sighted policy, and so on, attempting, by means of a carefully-selected epithet, to assume the point at issue, or at least to create an unfair prejudice in the mind of the hearer or reader whom we address.

freedom of speech must always be, on the whole, advantageous to the State; for it is highly conducive to the interests of the community that each individual should enjoy a liberty, perfectly unlimited, of expressing his sentiments."'

[4] It has sometimes been maintained that every syllogism is a petitio principii. Of course the controversy entirely turns upon the meaning of the terms, but, according to the account I have given of the two, a syllogism is so far from being a petitio principii, that every petitio principii is a distinct breach of the laws of syllogism. The conclusion of a syllogism is indeed implied by the two premisses taken in combination, and is, in fact, the compression into a single proposition of what, as premisses, were two distinct propositions, but, in a petitio principii, the conclusion is merely a re-assertion of one of the premisses; in the simpler cases, of a premiss in the same syllogism; in the argument in a circle, of a premiss in one of the preceding syllogisms of the series.

§ 4. III. The fallacy of Irrelevancy (or, as it is sometimes called, shifting ground) is technically termed *Ignoratio Elenchi*, i. e. ignorance of the syllogism required for the refutation of an adversary. Thus, in the strictest sense of the words, ignoratio elenchi is committed by a person who in a disputation does not confine himself to proving the contradictory or contrary of his adversary's assertion, or who proves a proposition other than the contradictory or contrary. But, like many other terms borrowed from the dialectical disputations of the ancients, this has now received a wider meaning. Whenever an argument is irrelevant to the object which a speaker or writer professes to have in view, it is called an ignoratio elenchi. Thus, if I am endeavouring to convince a person that some particular measure is for his personal interest, and I adduce arguments to prove that it contributes to the general utility, or that it is the necessary consequence of other acts of legislation, I am guilty of an ignoratio elenchi, as I should also be if, when it was my object to establish either of the other two conclusions, I were to appeal to his personal interest. When the question at issue is the truth of an opinion, it is an ignoratio elenchi to attack it for its novelty, or for its coming from a foreign source, or for any supposed consequences which may result from it, or to try to throw discredit on its author by saying that it has often been started before, and so is no discovery of his.

This fallacy is more common in spoken addresses than in books, as the feelings both of speaker and hearers

being more excited, and their judgment less critical, they are less likely to insist on relevancy of argument. On such occasions it most commonly takes the form of an *argumentum ad hominem*, whereby the speaker, in support of the truth of his assertions, or to throw discredit on an adversary, appeals, not to the unbiassed judgment of his auditors, but to their passions, interests, prejudices, sentiments and associations. The argumentum ad hominem, however, is not confined to set speeches; it sometimes occurs in writings, and frequently in debates. In the latter, it often assumes the shape of an appeal to the previous acts, or the previously expressed convictions of the opponent; 'That measure, or that argument, or that proposal does not come well from you, who once proposed such a measure, or expressed such an opinion, or advanced such an argument, or did such and such acts.' There are occasions when the argumentum ad hominem may legitimately be used as a retort, but it must be advanced as such, and not as an argument. It is so called in opposition to the argumentum *ad rem* or *ad judicium*. Similar phrases are used to express other forms of the ignoratio elenchi, as e. g. the *argumentum ad verecundiam*, *argumentum ad baculum*, &c. The *argumentum ad populum* I have treated as identical with the *argumentum ad hominem;* if called on to distinguish them, which seems unnecessary, I should refer the first to addresses made in the presence of a large auditory, the second to disputations with one or a few individuals[5].

[5] The student will find some amusing examples of ignoratio

§ 5. IV. The fallacy originating in ambiguity of language I noticed when warning the student against the employment of equivocal terms. This fallacy (whether we call it that of equivocal terms, of ambiguous terms, or of ambiguity of language) of course includes fallacies arising from any ambiguity which may attach to the quantity of the subject, as e. g. the fallacy arising from the ambiguous use of the word 'all,' which will be noticed below.

I now proceed to notice one or two common cases of this fallacy. The same term may often be used in one place distributively and in another collectively, and we may argue as if the term in both places had the same meaning. This is called the fallacy of *Composition* or *Division;* of composition, if we argue from a term taken distributively as if it were taken collectively; of division, if we argue from a term taken collectively as if it were taken distributively. Thus (to give common instances) 7 and 2 are (distributively) odd and even, nine is 7 and 2 (collectively); ∴ nine is odd and even. Here we argue from 7 and 2 taken distributively, as if they had been taken collectively, and the fallacy is one of composition. Five is one number, 3 and 2 (collectively) are five; ∴ 3 and 2 (distributively) are one number. Here the fallacy is one of division. Again, The people of England have a prejudice against the French, he is one of the people of England; ∴ he has a prejudice

elenchi, or irrelevant argument, in Sydney Smith's well-known jeu d'esprit, the Noodle's Oration.

against the French. The major premiss might be quite true, and still the particular man spoken of might have a strong sympathy with the French, and be a warm admirer of their institutions. Here we argue from the term 'people,' taken collectively, as if it had a distributive signification and whatever were predicable of the English people might be predicated of every single individual amongst them; hence the fallacy is one of division. The last instance is an example of a very common source of deception. A certain people, corporation, or society, in its collective capacity, has certain characteristics, has performed certain acts, passed certain resolutions, or is known to have expressed certain sentiments; hence it is unreflectingly supposed that any particular individual belonging to the class has the same characteristics, participates in the same sentiments, and has joined in the same acts. In many cases, of course, he may be a strong dissentient, and may have actively opposed the measures adopted.

The ambiguous use of the word 'all' furnishes a good instance of the fallacies of composition and division. We may argue from 'all,' meaning all taken together, as if it meant all severally, and thus commit the fallacy of division; or from 'all,' meaning all severally, as if it meant all taken together, and thus commit the fallacy of composition. Thus, when I say, 'All these boxes weigh so much,' or 'All these men can eat so much,' I leave it doubtful whether I mean all taken together or all taken severally. The ambiguity may be removed by substituting

for the word 'all,' when used in a distributive sense, 'every,' and, when used in a collective sense, 'the whole of.'

Another pair of fallacies which falls under the head of 'ambiguous terms' is the pair known as the *Fallacia Accidentis* (or the *Fallacia a dicto simpliciter ad dictum secundum quid*) and the *Fallacia a dicto secundum quid ad dictum simpliciter*. In the first we argue from what is true as a general rule (i. e. unless there be some modifying circumstances) as if it were true under all circumstances; in the second from what is true under certain special circumstances as if it were true as a general rule. Thus a particular walk may be an agreeable one, but it does not follow that it would be so in wet or windy weather; plain-speaking, frugality, generosity, may all be virtues, but it does not follow that it would be virtuous to practise them on all possible occasions. Or, to take instances of the second fallacy, a political revolution may, under particular circumstances, be necessary to the welfare or existence of a country, but it does not follow that a state of society, in which political revolutions are frequent, is either necessary or desirable; it may be necessary if I am suffering from a particular disease that I should take opium or abstain from labour, but it does not follow that these practices would be good for me when I am restored to health. The fallacies are due to our not sufficiently qualifying the terms which we use, and, by insisting on precision of language, they may always be avoided.

Though the definitions I have given of this pair of fallacies are conformable to the usage of most modern logicians[6], and are stated in a form which is most likely to be of practical service to the student, they do not exactly correspond with the original meaning of the expressions. The 'Fallacia Accidentis' and the 'Fallacia a dicto secundum quid ad dictum simpliciter,' according to their original usage, applied to those cases in which a term, when not implying accidents, was confounded with the same term, when implying accidents. Thus, to take the common instance (which is sufficiently absurd): 'What you buy in the market you eat; raw meat is what you buy in the market; ∴ raw meat is what you eat.' Here it may be replied that what we buy in the market we do indeed eat, but not necessarily in the same state in which we buy it at market[7]. This particular instance is an

[6] As, for instance, Mill (*Logic*, Bk. V. ch. vi. § 4), *Port Royal Logic* (Part III. ch. xix. § 5, 7). The latter virtually treats both fallacies as if they were a dicto secundum quid ad dictum simpliciter.

[7] Mr. de Morgan adduces one of Boccaccio's stories as affording an amusing instance of the fallacia accidentis. It is the old example of the 'raw meat' in another form:—

'A servant who was roasting a stork for his master was prevailed upon by his sweetheart to cut off a leg for her to eat. When the bird came upon table, the master desired to know what was become of the other leg. The man answered that storks had never more than one leg. The master, very angry, but determined to strike his servant dumb before he punished him, took him next day into the fields where they saw storks, standing each on one leg, as storks do. The servant turned triumphantly to his master, on which the latter shouted, and the birds put down their other legs and flew away. "Ah, sir," said the servant, "you did not shout to the stork at dinner

example of the Fallacia Accidentis. From their technical meaning, these fallacies would easily pass into their present signification, which is both more intelligible and of greater practical service.

I may notice one more example of the errors due to ambiguous language, viz. the fallacy of what may be called *Paronymous Terms.* The same word may often assume different forms, as substantive, adjective, adverb, or verb, but it does not follow, when it has assumed these different forms, that they all retain corresponding meanings. It has been already noticed that the words probably, probable, probability, though the two last are themselves ambiguous, vary in meaning according as we use the adverb, the adjective, or the substantive. Thus, if I hear some one ask the question 'What is the probability of my throwing an ace with a die at a single throw?' I cannot infer that in any single throw I shall probably throw an ace. Again, because a man has done something unjust (i. e. has committed an act which in its results is unjust), I cannot infer that he has acted unjustly (i. e. with intentional injustice), nor, even if he has acted unjustly (i. e. in one or more instances), can I infer that he is an unjust man (i. e. a man of unjust habits or character). To take an old instance, because projectors are unfit to be trusted, and this man has formed a project, it does not follow that he is unfit to be trusted. Nor from the meaning attached to the expressions, kingly,

yesterday: if you had done so, he would have shewn his other leg too."'

nobly, gentlemanly, can we argue to the usual qualities of a king, a nobleman, or a gentleman; nor, on the meanings of the words 'to trow,' 'to represent,' can we base any sound argument as to the nature of truth, or the duties of a representative. All instances of this fallacy, when stated syllogistically, involve four terms, and so offend against the rules for the construction of a syllogism, but, as we do not ordinarily state our arguments in a syllogistic shape, and these fallacies undoubtedly impose on us through the ambiguity of language, it is better to consider them here rather than under the second head.

Many other forms of fallacy may be regarded as due to ambiguities of language, but it has perhaps been the tendency of modern logicians, and especially of Whately, to overload this division of fallacies, and to treat as merely differences of language what are in reality radical differences of opinion. At the same time it cannot be denied that terms expressive of fundamental conceptions in their several sciences, such as faith, church, election, law, loyalty, federation, justice, value, capital force, nature, natural, &c., are frequently used, in the same discussion, in the most widely divergent senses, and are consequently the source of endless confusion in our reasonings. Thus the term 'faith' may mean either a *belief* in certain propositions, or *confidence*, trust, and repose in a certain person; the word 'church' may mean the whole body of Christians (and, of course, in this sense its signification will vary according to the meaning attached to the term Christian), a

particular section of Christians, a congregation meeting in a certain place, the place of meeting, and, lastly, by a strange perversion of the term, the clergy as distinguished from the laity; the term 'loyalty' may mean either attachment to the laws of a country in general, special attachment to some particular portion of the laws, or, in its most restricted sense, personal attachment to the supreme ruler; 'capital' may mean either the amount of money possessed by a trader, or his whole stock of commodities available for future production; 'natural' may express either the original condition of a thing, or the state into which it is ultimately developed, besides having countless other meanings. On account of the various significations which may be attached to the same term, it is necessary, in entering on any investigation, carefully to define the terms to be employed, and never, without express notice, to deviate from the sense thus imposed upon them [8].

[8] The so-called 'Fallacia plurium interrogationum,' has not been noticed in the text, because it is a rhetorical artifice, rather than a logical fallacy. It consists in covertly putting as a single question what is in reality two, as for instance, 'Are gall and honey sweet?' 'Have you cast your horns?' (known as 'cornutus'). 'What did you take, when you broke into my house last night?' 'Have you given up beating your father?' The object is to entrap the respondent into an admission which he would otherwise not be likely to make.

CHAPTER IX.

On Method as applied to the arrangement of Syllogisms in a Train of Reasoning.

I DO not propose to treat of Method in general (for this would involve a discussion of induction and the various relations in which it stands to deductive inference), but it may be useful to the student if I offer a few remarks on Method under the limitation stated in the heading of this chapter. When syllogisms are combined in a train of reasoning, we may either commence with the conclusion, and ask what reasons we have for believing it, and then go on to ask the reason for believing the premisses, and so on, till at last we arrive at some propositions of which there is no doubt, or in which we at least can acquiesce; or else we may follow the reverse process, and commencing with propositions which are the result of some previous investigation, or which we at all events accept as true, may go on combining them with each other, till at last we arrive at some conclusion which we regard as sufficiently important to terminate our enquiries. The former method will be familiar to my readers as that by which we solve what are called 'geometrical deductions,' and in fact as the method which we generally though not

universally employ when we are attempting to resolve difficulties for ourselves; the latter as the method by which the propositions in Euclid are proved, and in fact as the method which we generally though not universally employ, when it is our object to teach others, either orally or by book. Now the former method is called *Analytical* (from the Greek word ἀνάλυσις), because it may be regarded as the breaking up of a whole into its parts, the resolution of the final conclusion of a series of syllogisms into the various premisses on which it depends, and of which it is, as it were, the total expression. The latter method is called *Synthetical* (from the Greek word σύνθεσις), because it may be regarded as the putting together of the parts into a whole, the combination of the various premisses into a conclusion which is, as it were, their total result. The Synthetical Method is also sometimes called *Progressive*, and the Analytical Method *Regressive*, for reasons which will be apparent from what has already been said.

The words *a priori* and *a posteriori* may also be used to express the same distinction. In inductive inference (to which these terms are more properly applied) we are said to proceed *a posteriori*, when, a certain event having taken place, we attempt to trace the steps by which it came about, or, a certain phenomenon being presented to us for examination, we attempt to infer the mode of its production; and, vice versâ, we are said to proceed *a priori*, when, from our knowledge of certain circumstances, we attempt to predict an event, or, by putting in

operation certain causes, we attempt to discover their effect. Similarly, in deductive inference, if, a conclusion being assumed as provisionally true, we attempt to discover reasons for it, we may be said to proceed *a posteriori;* if, starting with the premisses, we go on combining them to see whither they will lead us, we may be said to proceed *a priori.* In the former method of reasoning, we are peculiarly liable to impose on ourselves or others by availing ourselves of premisses which are fanciful, obscure, incapable of proof, questionable, or untrue, especially if the conclusion express some cherished conviction or some position which it is the interest of ourselves, our class, or our party to accept and to disseminate. Whenever, therefore, we argue from our conclusions backwards, especial caution is required, if it be our sincere desire to test our convictions impartially.

There is also a more general employment of these latter expressions, according to which the term *a posteriori* is appropriated to designate inductive reasoning, which ought always to be based on individual facts of observation; *a priori* to designate deductive reasoning, which proceeds from general principles. The expression *a priori* is often applied by way of censure to deductive reasoning, when the general principles from which it proceeds are supposed to rest on no ultimate inductions from fact, but to be mere assumptions arbitrarily taken for granted by the author who employs them.

Note.—For an account of the various senses in which the words 'analysis' and 'synthesis' are or have been employed, the student is referred to Sir W. Hamilton's *Lectures on Logic*, Lect. xxiv, and Dr. Mansel's edition of *Aldrich*, Appendix G.

APPENDIX.

On the Five Words 'Genus,' 'Species,' 'Differentia,' 'Property,' 'Accident.'

WHEN two classes are so related to each other that one is contained under the other, the larger or containing class is called the *Genus*, and the smaller or contained class is called the *Species*. Thus, if we compare animal and man, man is a species, and animal the genus; if a man and Englishman, man is the genus, and Englishman a species: if we compare moss-rose and rose, moss-rose is a species, and rose the genus; if rose and flower, flower is the genus, and rose a species.

Genera and Species are denoted by Common Terms, which themselves also, as well as the groups which they denote, are called Genera and Species.

In the A proposition, a genus can be predicated of a species, but not a species of a genus. Thus we can say 'All men are animals,' or 'Man is an animal,' but not 'All animals are men,' or 'An animal is a man.' In the I proposition, on the other hand, the genus may be the subject, and in the O proposition, if the terms compared are genus and species, must be.

The distinction between Differentia, Property, and Accident is more difficult, but may be explained by reference to what has been said (Pt. I. ch. ii.) on the Connotation of Terms.

As the former distinction was applicable to classes and the common terms which denote them, so this is applicable to

attributes and the attributives by which attributes are expressed. It may be noticed also that a Differentia, Property, or Accident, being expressed by an Attributive, must be a predicate and cannot be a subject. Now, taking any common term, or any abstract term used as a common term, like triangle, an attributive predicated of it may express either part of its connotation or not. Thus, if we assert that 'A triangle is a three-sided rectilineal figure,' the term 'three-sided,' like the genus 'figure,' is connoted by the very term 'triangle.' Moreover, the term 'three-sided' serves to *differentiate* or distinguish 'triangle' from all other rectilineal figures, such as quadrilateral, pentagon, &c. Hence we may define differentia as on p. 45. Here however a difficulty occurs. Two or more species falling under the same genus may be distinguished by more than one differencing attribute. In this case we should speak of the *differentiæ*, or we might speak of the *differentia* as the sum of the *differentiæ*.

But, in case the attributive does not express any part of the connotation of the term, it may, nevertheless, express some attribute which follows from the connotation. Thus, if we say 'A triangle is a rectilineal figure having the sum of its angles equal to two right angles,' we are predicating by the expression 'having' &c., not part of the connotation of the word triangle, but an attribute which may be directly inferred from part of the connotation. Or, again, if we were dealing with some moral or physical phenomenon, the attribute might follow, not as a conclusion from a premiss, but as an effect from a cause. Thus we might predicate of man that he is 'capable of progress,' this capability being regarded as an effect of his rationality; or of animal and vegetable tissues that they are liable to decay, this liability being regarded as an effect of the material of which they are composed and the influences to which they are subject. Hence the definition of *Property* on p. 45.

Lastly, the attributive may neither express any part of the connotation of the term, nor any attribute which follows from part of the connotation. In this case it is called an *Accident.* But accidents are of two kinds. If an accident may be predicated of all the individuals denoted by a common term, as, to take the conventional instance, blackness of crows, it is called an *Inseparable Accident.* In the most common case, where it is predicable of some of the individuals and not of others, as, for instance, blackness of men, it is called a *Separable Accident.*

The student is recommended to compare, throughout, this explanation with the definitions given on p. 45.

EXAMPLES.

EXAMPLES.

EXAMINE the following Definitions or Descriptions (pointing out their faults, if any, and, where they are sufficient, stating under what head of Definition or Description they fall).

(1) A square is a four-sided rectilinear figure, having all its sides equal.

(2) Monarchy is a form of political government in which one man is sovereign.

(3) An University is a corporation which grants learned degrees.

(4) Logic is the Art of Reasoning.

(5) A plane triangle is a figure generated by the section of a cone passing through the vertex and perpendicular to the base.

(6) Wealth is the sum of things useful, necessary, and agreeable.

(7) Man is a mammal having hands and cooking his own food.

(8) An animal is a sentient organised being.

(9) A liquid is that which can be poured out.

(10) A Federation is a political union the members of which are bound together for purposes of offence and defence.

(11) Man is a mammal possessing the power of articulate speech.

(12) Political Philosophy is the science of the laws which govern the equilibrium and development of human society.

Examine the following Divisions, substituting, where they are incorrect, one or more correct ones.

(1) Men into Aryans, Mongolians, Africans, and Americans.
(2) Quadrilateral Figures into Squares, Rectangles, Parallelograms, and Rhomboids.
(3) The Fine Arts into Painting, Drawing, Sculpture, Architecture, Poetry, and Photography.
(4) Governments into Monarchies, Tyrannies, Oligarchies, and Democracies.
(5) Books into entertaining and unentertaining.
(6) Men into those who lend and those who borrow.
(7) The Sciences into Physical, Social, Ethical, Logical, and Metaphysical.
(8) Plants into Flowering Plants, Mosses, Ferns, and Pines.
(9) The origin of Colonies is to be traced either to the necessity for frontier garrisons (as amongst the Romans) or to the poverty or discontent of the inhabitants of the mother-country (as amongst the Greeks and ourselves).
(10) 'The general stock of any country or society is the same with that of all its inhabitants or members, and therefore naturally divides itself into the same three portions, each of which has a distinct function or office:

1st, that portion which is reserved for immediate consumption, and of which the characteristic is that it affords no revenue or profit;

2nd, the fixed capital, of which the characteristic is that it affords a revenue or profit without circulating or changing masters;

3rd, the circulating capital, of which the characteristic is that it affords a revenue only by circulating or changing masters.'—Adam Smith's *Wealth of Nations*, Vol. ii. ch. i.

Convert the following propositions (previously permuting them, where necessary).

(1) All plane triangles are rectilinear figures.
(2) All plane triangles are three-sided rectilinear figures.
(3) All plane triangles may be defined as three-sided rectilinear figures.
(4) Some branches of Mathematics admit of a direct practical application.
(5) Men of fair promises are often not to be trusted.
(6) Some members of the Government are not prepared to accept the measure.
(7) Virtue is a condition of Happiness.
(8) Virtue is the condition of Happiness.
(9) A syllogism is a form of inference.
(10) With man many things are impossible.
(11) Some men of great powers of imagination are not poets.
(12) None but persons of great powers of imagination are poets.
(13) What I have written, I have written.
(14) Propositions are either simple or complex.
(15) The proper study of mankind is man.
(16) Only the ignorant affect to despise knowledge.
(17) He can't be wrong whose life is in the right.

State in logical form (where necessary) and examine the following arguments.

(1) Every book is liable to error,
Every book is a human production;
∴ All human productions are liable to error.

(2) All tulips are beautiful flowers,
No roses are tulips;
∴ No roses are beautiful flowers.

(3) Some men are wise,
Some men are good;
∴ Some wise men are good.

(4) All wise men are good,
Socrates was a wise man;
∴ He was good.

(5) Some mathematicians are logicians,
No logicians are unacquainted with the works of Aristotle;
∴ Some mathematicians are not unacquainted with the works of Aristotle.

(6) No persons destitute of imagination are true poets,
Some persons destitute of imagination are good logicians;
∴ Some good logicians are not true poets.

(7) No persons destitute of imagination are true poets,
Some persons destitute of imagination are good logicians;
∴ Some true poets are not good logicians.

(8) If Cæsar was a tyrant, he deserved to die,
Cæsar was not a tyrant;
∴ He did not deserve to die.

(9) If virtue is involuntary, vice is also involuntary,
Vice is voluntary;
∴ Virtue is also voluntary.

(10) All valid syllogisms have three terms,
This syllogism has three terms;
∴ It is a valid syllogism.

(11) Some learned men have become mad,
He is not a learned man;
∴ He will not become mad.

(12) The Reformers were bitter enemies of the Papal Supremacy,

A B was a Reformer (for he was prominent in supporting the Reform Bill of 1832);
∴ He was a bitter enemy of the Papal Supremacy.

(13) Logic is either a science or an art,
It is a science;
∴ It is not an art.

(14) Six and seven are even and uneven,
Thirteen is six and seven;
∴ Thirteen is even and uneven.

(15) He must be a Mahommedan, for only Mahommedans hold these opinions.

(16) He must be a Mahommedan, for all Mahommedans hold these opinions.

(17) To reject this proposal would be unreasonable, and consequently to accept it is reasonable.

(18) This event happened either at Rome, Naples, or Florence; it did not happen at Rome or Naples, and consequently it must have happened at Florence.

(19) Logic is indeed worthy of being cultivated, if Aristotle is to be regarded as infallible; but he is not: Logic therefore is not worthy of being cultivated.

(20) An indissoluble association of ideas commands belief, and consequently every belief is the consequence of an indissoluble association of ideas.

(21) This measure would be destructive of the national prosperity, and I cannot adduce a more cogent argument than that, five years ago, you were yourself of the same opinion.

(22) If a man cannot make progress towards perfection, he must either be a brute or a divinity; but no man is either: therefore every man is capable of such progress.

(23) Lias lies above New Red Sandstone, New Red Sand-

stone lies above coal; therefore Lias lies above Coal.

(24) My hand touches the pen; the pen touches the paper: therefore my hand touches the paper.

(25) A, B, C, D, and E, are the only German students I know: they are all men of considerable intellectual attainments, and consequently I may infer that all German students are men of considerable intellectual attainments.

(26) All equilateral triangles are equiangular, and therefore all equiangular triangles must be equilateral.

(27) These two figures are equal to the same figure, and therefore to one another.

(28) For those who are bent on cultivating their minds by diligent study, the incitement of academical honours is unnecessary; and it is ineffectual for the idle and such as are indifferent to mental improvement: therefore the incitement of academical honours is either unnecessary or ineffectual.

(29) He has no appreciation of beauty, for he has no taste for Pictures.

(30) Warm countries alone produce wines, Spain is a warm country; therefore Spain produces wines.

(31) The Germans are a literary nation; therefore A B, who is a German, is a literary man.

(32) We must increase the income-tax, for war has become a necessity, and we cannot go to war without money, and money can only be raised by taxation, and the only tax which the resources of the country can bear is the income-tax, which will fall on the richer part of the population.

(33) Governors of dependencies should be vested with absolute power, for otherwise it is impossible to crush rebellion.

(34) A is larger than B, and B is larger than C; therefore, a fortiori, A is larger than C.

(35) Alexander is the son of Philip, and therefore Philip is the father of Alexander.

(36) If 'to improve is to change, and to be perfect is to have changed often,' what hope can we entertain of those who oppose change?

(37) Luxury is at once beneficial and injurious to society; for luxury is the using the gifts of Providence, either to the injury of the user, or to the injury of others towards whom the user stands in any relation which obliges him to aid and assistance; but luxury causes expenditure of money, and therefore is beneficial to society.

(38) Old age is wiser than youth; therefore it is only reasonable that we should be guided by the decisions of our ancestors.

(39) A man cannot always be right in his opinions, and therefore we ought continually to distrust our judgments.

(40) Had an armistice been beneficial to France and Germany, it would have been agreed upon by those powers; but such has not been the case: it is plain, therefore, that an armistice would not have been advantageous to either of the belligerents.

(41) If education is popular, compulsion is unnecessary; if unpopular, compulsion will not be tolerated.

(42) I will not do this act, because it is unjust; I know that it is unjust, because my conscience tells me so, and my conscience tells me so, because the act is wrong.

(43) This proposition is too good to be practicable.

(44) Such and such a system of education has produced several distinguished men; therefore it does not admit of any improvement.

(45) Slavery is a natural institution ; but what is natural is just, and what is just it is unjust to abrogate ; and consequently it would be unjust to abrogate slavery.

(46) 'Mercy but murders, pardoning those that kill.'

Romeo and Juliet, Act iii. Sc. 1.

(47) A, B, and C have distinguished themselves both in athletic sports and intellectual pursuits ; therefore those who are most famous for their excellence in athletic sports are generally most famous for their intellectual attainments as well.

(48) What is called a 'constitutional monarchy' is impossible, for the sovereign authority of a state cannot be limited by law ; now the king is sovereign, and, as such, cannot be subject to any superior authority.

(49) Parallel lines are equi-distant ; for, if from two points in one of them perpendiculars be drawn to the other, these perpendiculars are parallel (Euc. I. 28), and the two lines intercepted between them are parallel ; therefore a parallelogram is formed, of which the perpendiculars are opposite sides and therefore equal.

(50) If Bacon's opinion is right, it is improper to stock a new colony with the refuse of jails : but this course we must allow not to be improper, if our method of colonising New South Wales be a wise one : if this be wise, therefore, Bacon's opinion is not right.

(51) 'Profit' is interpreted in the dictionary 'advantage' ; to take profit, then, is to take advantage : it is wrong to take advantage of one's neighbour : therefore it is wrong to take profit.

(52) Romulus must be a historical person, because it is not at all likely that the Romans, whose memory was only burdened with seven kings, should have forgotten the most famous of them, namely, the first.

(53) You maintain that an action can only be called virtuous,

if it contribute to the welfare of mankind or of some section of mankind: hence you are bound to regard every convenient object, for instance a horse, a tree, or a chair, as virtuous.

(54) The knowledge of things is more useful than the knowledge of words; and, therefore, the study of nature is more improving to the mind than the study of language.

(55) In a lottery it is improbable that any particular ticket-holder will draw the prize. But some one must draw the prize. Therefore something improbable must happen.

(56) My informant A heard his story from B, who would certainly tell it as originally told to him; B heard it from C, who would probably tell it accurately; C from D, who would also probably tell it accurately; D from E, who, I have no reason to suppose, would tell it inaccurately: I may consequently receive A's story as probably accurate.

(57) Large colonies are as detrimental to the power of a state as overgrown limbs to the vigour of the human body.

(58) All law is an abridgment of liberty, and consequently of happiness.

(59) I am under an obligation to do it, but he who is obliged has no power of resistance; consequently I have no choice about the matter.

(60) You never give an opinion without believing yourself to be right, and therefore you must suppose yourself to be infallible.

(61) If man be not a necessary agent, determined by pleasure and pain, there is no foundation for rewards and punishments. These would be useless, unless men were necessary agents, and were determined by pleasure and pain: because, if men were free and

indifferent to pleasure and pain, pain could be no motive to cause men to observe the law.

(62) Night invariably precedes day, and therefore night must be the cause of day.

(63) The planet Mars resembles the Earth in the possession of an atmosphere, clouds, and water, and has a temperature in which terrestrial life might exist, and, therefore, it is probably inhabited as the earth is.

(64) 'If it be fated that you recover from your present disease, you will recover, whether you call in a doctor or not; again, if it be fated that you do not recover from your present disease, you will not recover, whether you call in a doctor or not: but one or other of these contradictories is fated, and therefore it can be of no service to call in a doctor.'—(*Ignava Ratio.*)

(65) The story of the formation of the human race by Prometheus must be true, for the clay from which he formed it was shown in Greece within historical times.

(66) The Latin word 'virtus' originally meant 'manliness'; hence the virtue of manliness or courage is the highest virtue and the type of all other virtues.

(67) This person may reasonably be supposed to have committed the theft, for he can give no satisfactory account of himself on the night of the alleged offence; moreover he is a person of bad character, and, being poor, is naturally liable to a temptation to steal.

(68) Opium produces sleep, for it possesses a soporific virtue.

(69) The student of History is compelled to admit the truth of the Law of Progress, for he finds that Society has never stood still.

(70) You are inconsistent with yourself, for you told me yesterday that there was a presumption of this man's guilt, and now, when I say that I may presume his guilt, you contradict me.

71) 'Suppose one man should by fraud or violence take from

another the fruit of his labour with intent to give it to a third, who, he thought, would have as much pleasure from it as would balance the pleasures which the first possessor would have had in the enjoyment and his vexation in the loss of it; suppose also that no bad consequences would follow; yet such an action would surely be vicious.' Butler, *On the Nature of Virtue.*

(72) There exist many differences of opinion and much uncertainty with regard to many questions connected with Geology; consequently Geology is not a science, and any arguments which assume the truth of geological theories must invariably be regarded with considerable suspicion.

(73) 'Suppose Achilles to move ten times as fast as the Tortoise, but the Tortoise to have the start of Achilles, say, by one-tenth of the distance to be traversed; when Achilles has arrived at the point from which the Tortoise started, the Tortoise will still be one-hundredth part of the whole distance in advance of him; when Achilles has reached this point, the Tortoise will still be one-thousandth part of the whole distance in advance of him; and so on. Thus Achilles will never be able to pass the Tortoise.'

(*Fallacy of Achilles and the Tortoise.*)

(74) 'Epimenides the Cretan says that "all the Cretans are liars," but Epimenides is himself a Cretan; therefore he is himself a liar. But if he be a liar, what he says is untrue, and consequently the Cretans are veracious; but Epimenides is a Cretan, and therefore what he says is true; hence the Cretans are liars, Epimenides is himself a liar, and what he says is untrue. Thus we may go on alternately proving that Epimenides and the Cretans are truthful and untruthful.'—(*Fallacy of Mentiens.*)

(75) The idea of the obligation to virtue is innate, for it is

found in all men, and it could not be universal if it were acquired by experience.

(76) Berkeley's Theory of the Non-existence of Matter is palpably absurd, for it is impossible even to place one's foot on the ground without experiencing the resistance of matter.

(77) I have no hesitation in saying that the proposition, however good in theory, is in practice utterly absurd.

(78) If any objection that can be urged would justify a change of established laws, no laws could reasonably be maintained; but some laws can reasonably be maintained: therefore no objection that can be urged will justify a change of established laws.

(79) I cannot accept your opinion as true, for it seems to me that its general recognition would be attended with the most injurious consequences to society.

(80) Why should any but professional moralists trouble themselves with the solution of moral difficulties? For, as we resort to a physician in case of any physical disease, so, in the case of any moral doubt or any moral disorganisation, it seems natural that we should rely on the judgment of some man specially skilled in the treatment of such subjects.

(81) 'Wood, stones, fire, water, flesh, iron, and the like things, which I name and discourse of, are things that I know. And I should not have known them but that I perceived them by my senses; and things perceived by the senses are immediately perceived; and things immediately perceived are ideas; and ideas cannot exist without the mind; their existence therefore consists in being perceived; when, therefore, they are actually perceived there can be no doubt of their existence.'

Berkeley, *Third Dialogue between Hylas and Philonous.*

(82) 'And because the greatest part of men are such as prefer their own private good before all things, even

that good which is sensual before whatsoever is most divine; and for that the labour of doing good, together with the pleasure arising from the contrary, doth make men for the most part slower to the one and proner to the other, than that duty prescribed them by law can prevail sufficiently with them: therefore unto laws that men do make for the benefit of men it hath seemed always needful to add rewards, which may more allure unto good than any hardness deterreth from it, and punishments, which may more deter from evil than any sweetness thereto allureth.'

Hooker, *Eccl. Pol.* Bk. I. x. (6.)

(83) 'The scarcity of a dear year, by diminishing the demand for labour, tends to lower its price, as the high price of provisions tends to raise it. The plenty of a cheap year, on the contrary, by increasing the demand, tends to raise the price of labour, as the cheapness of provisions tends to lower it. In the ordinary variations of the price of provisions, those two opposite causes seem to counterbalance one another; which is probably in part the reason why the wages of labour are everywhere so much more steady and permanent than the price of provisions.'

Adam Smith, *Wealth of Nations;* Bk. I. ch. viii.

(84) 'I am a Jew. Hath not a Jew eyes? hath not a Jew hands, organs, dimensions, senses, affections, passions? fed with the same food, hurt with the same weapons, subject to the same diseases, healed by the same means, warmed and cooled by the same winter and summer, as a Christian is? If you prick us, do we not bleed? If you tickle us, do we not laugh? If you poison us, do we not die? and if you wrong us, shall we not revenge? If we are like you in the rest, we will resemble you in that.'

Merchant of Venice, Act iii. Sc. 1.

(85) 'The most striking and important of the effects of heat consist, however, in the liquefaction of solid substances, and the conversion of the liquids so produced into vapour. There is no solid substance known which, by a sufficiently intense heat, may not be melted, and finally dissipated in vapour; and this analogy is so extensive and cogent, that we cannot but suppose that all those bodies which are liquid under ordinary circumstances, owe their liquidity to heat, and would freeze or become solid if their heat could be sufficiently reduced. In many we see this to be the case in ordinary winters; for some, severe frosts are requisite; others freeze only with the most intense artificial colds; and some have hitherto resisted all our endeavours; yet the number of these last is few, and they will probably cease to be exceptions as our means of producing cold become enlarged. A similar analogy leads us to conclude that all aëriform fluids are merely liquids kept in the state of vapour by heat. Many of them have been actually condensed into the liquid state by cold accompanied with violent pressure; and as our means of applying these causes of condensation have improved, more and more refractory ones have successively yielded. Hence we are fairly entitled to extend our conclusion to those which we have not yet been able to succeed with; and thus we are led to regard it as a general fact, that the liquid and aëriform or vaporous states are entirely dependent on *heat*, that were it not for this cause, there would be nothing but solids in nature; and that, on the other hand, nothing but a sufficient intensity of heat is requisite to destroy the cohesion of every substance, and reduce all bodies, first to liquids, and then into vapour.'

Herschel, *On the Study of Natural Philosophy*.

(86) 'We are not inclined to ascribe much practical value to

that analysis of the inductive method which Bacon has given in the second book of the Novum Organum. It is indeed an elaborate and correct analysis. But it is an analysis of that which we are all doing from morning to night, and which we continue to do even in our dreams.' Macaulay, *Essay on Bacon.*

(87) 'Promises are not binding where the performance is *unlawful.* There are two cases of this; one, where the unlawfulness is known to the parties, at the time of making the promise; as where an assassin promises his employer to despatch his rival or his enemy; or a servant to betray his master. The parties in these cases are not obliged to perform what the promise requires, *because they were under a prior obligation to the contrary.* From which prior obligation what is there to discharge them? Their promise, their own act and deed. But an obligation, from which a man can discharge himself by his own act, is no obligation at all. The guilt therefore of such promises lies in the making, not in the breaking of them; and if, in the interval betwixt the promise and the performance, a man so far recover his reflection, as to repent of his engagements, he ought certainly to break through them.'

Paley, *Moral and Political Philosophy,*
Bk. III. Part I. ch. v.

(88) 'This, I think, any one may observe in himself, and others, that the *greater visible Good* does not always raise men's *desires* in proportion to the greatness, it appears, and is acknowledged to have: Though every little Trouble moves us, and sets us on work to get rid of it. The Reason whereof is evident from the Nature of our *Happiness* and *Misery* itself. All present Pain, whatever it be, makes a part of our present *Misery:* But all absent Good does not at any time

make a necessary part of our present *Happiness*, nor the absence of it make a part of our *Misery*. If it did, we should be constantly and infinitely miserable; there being infinite degrees of Happiness, which are not in our possession. All *Uneasiness* therefore being removed, a moderate portion of Good serves at present to content men; and some few degrees of pleasure in a succession of ordinary Enjoyments make up a Happiness, wherein they can be satisfied. If this were not so, there could be no room for those indifferent, and visibly trifling Actions, to which our *Wills* are so often determined; and wherein we voluntarily waste so much of our Lives; which remissness could by no means consist with a constant determination of *Will* or *Desire* to the greatest apparent Good.'

Locke, *Essay concerning Human Understanding*, Bk. II. ch. xxi. § 44.

(89) 'There is only one part of the Protectionist scheme which requires any further notice: its policy towards colonies, and foreign dependencies; that of compelling them to trade exclusively with the dominant country. A country which thus secures to itself an extra foreign demand for its commodities, undoubtedly gives itself some advantage in the distribution of the general gains of the commercial world. Since, however, it causes the industry and capital of the colony to be diverted from channels, which are proved to be the most productive, inasmuch as they are those into which industry and capital spontaneously tend to flow; there is a loss, on the whole, to the productive powers of the world, and the mother country does not gain so much as she makes the colony lose. If, therefore, the mother country refuses to acknowledge any reciprocity of obligation, she imposes a tribute on the colony in an indirect mode, greatly more oppressive and in-

jurious than the direct. But if, with a more equitable spirit, she submits herself to corresponding restrictions for the benefit of the colony, the result of the whole transaction is the ridiculous one, that each party loses much, in order that the other may gain a little.'

Mill's *Political Economy*, Bk. V. ch. x. § 1.

(90) 'The money to replace what has been burned will not be sent abroad to enrich foreign manufactures; but, thanks to the wise policy of protection which has built up American industries, it will stimulate our own manufactures, set our mills running faster, and give employment to thousands of idle workmen. Thus in a short time our abundant natural resources will restore what has been lost, and in converting the raw material our manufacturing interests will take on a new activity.'

New York Tribune of Oct. 24, 1871, quoted by Professor Caimes in 'Some Leading Principles of Political Economy.'

*** A syllogism, when not stated in logical form, usually appears to contain four, or perhaps more, terms. We must not, however, reject it on that account, till we have previously attempted to translate the propositions of which it is composed into equivalent propositions, containing among them three terms only. The cases in which this cannot be done are either those in which the terms of the conclusion, or at least one of them, are distinct, not in form only, but in meaning, from any of the terms employed in the premisses, or those in which there is no term common, or capable of being represented as common, to the two premisses. Thus the syllogism 'Lias lies above New Red Sandstone, New Red

Sandstone lies above Coal; therefore Lias lies above Coal,' obviously admits of being stated in the form, 'Whatever lies above New Red Sandstone lies above Coal, Lias lies above New Red Sandstone; therefore Lias lies above Coal,' and consequently ought not to be rejected as having four terms. But the premisses 'Lias lies above New Red Sandstone, the Cretaceous System lies above the Oolitic,' contain no term common, nor any term capable (from a mere inspection of the language) of being represented as common, to the premisses, and hence they might fairly be rejected as containing four terms, and consequently leading to no conclusion. Even here, however, by any one who possessed sufficient special knowledge of Geology to be aware that the Oolitic System lies above Lias, the premisses might be represented as containing three terms only, and as necessitating the conclusion 'The Cretaceous System lies above New Red Sandstone.' But from such premisses as 'Lias lies above New Red Sandstone,' 'A Painting should represent beauty of colour as well as beauty of form,' no conclusion whatever could be drawn by means of either logical or special knowledge. The premisses are, in fact, utterly alien to each other, or, in other words, they are not *in pari materiâ*. In examining an argument, the student may always avail himself of any special knowledge which he may possess, provided that, in his answer, he carefully distinguish between what is due to such special knowledge and what to a knowledge of the ordinary usages of language and of the rules of Logic.

In the first example given above, a mere knowledge of the rules of Logic, without any reference whatever to the usages of language, would not justify us in drawing any conclusion from the premisses; the slightest acquaintance with the usages of language would however enable us to represent the terms, which are apparently four, as three, and to infer the conclusion 'Lias lies above Coal.' But in the second example no acquaintance either with the rules of Logic or with the ordinary usages of language would enable us to draw the conclusion, 'The Cretaceous System lies above New Red Sandstone;' such a conclusion, though valid, is only justified by a special knowledge of Geology: no person, unacquainted with the facts of Geology, would be justified in admitting the conclusion as an inference from the premisses.

Where it is obvious that an argument is intended to be syllogistic, and only one premiss is stated, it is of course expected that the student will supply the other premiss.

In some of the examples, it may be an useful exercise to discuss the truth of the premisses as well as the legitimacy of the conclusion.

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